UNIVERSITY OF ALBERTA

AN ANALOG SIMULATION OF THE FARM TRACTOR SUSPENSION SYSTEM UNDER ROAD TRAVEL

by

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled "An Analog Simulation of the Farm Tractor Suspension System Under Road Travel" submitted by Lal N. Shukla in partial fulfilment of the requirements for the degree of Master of Science.

ABSTRACT

The objective of this thesis was to compare the effects of zero ballast, liquid ballast and *lead ballast on the stability of a farm tractor. The equations representing the dynamic forces acting on farm tractors were simulated on an analog computer and the effects of speed, slope and centrifugal force on the stability of a farm tractor were then studied. The computer output was recorded and graphs of the output can be seen in Appendix I. Experiments were also conducted to see the effect on static spring rate of tractor tires when the tires were resting on level ground and when they were on 6" x 6", 6" x 4", and 6" x 2" wooden blocks, all 3 5/8" high.

The results of this project were as follows:-

1. Lead ballast tires bounced the most and zero ballast tires bounced the least.
2. Lead ballast tires had the highest natural frequency and 100% air filled tires had the lowest natural frequency.
3. Static spring rate depends upon the length of tire contact on the supporting surface.
4. A tractor on 75% liquid filled tires has better roll stability than a tractor on 100% air filled tires.
5. A tractor on lead ballast tires had greater stability than on 75% liquid filled or 100% air filled tires when operated on level ground. When operating on sloping ground or turning the tractor was less stable with lead ballast tires than with the other two ballasts under similar conditions.

* As used here lead ballast refers to the commercial product "led ballast".

6. Roll instability was greater at an operating speed of 7.5 mph than at a speed of 15 mph.
7. At higher speeds pitch instability was greater than at lower speeds.
8. There was greater pitch instability with lead ballast tires than with 75% liquid filled or 100% air filled tires.

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1. INTRODUCTION

1.1 PROBLEM AND REASON FOR SELECTING THE PROBLEM

Tractor sideways overturning is much more common and causes more fatalities than rearing in tractor accidents. A study of tractor accidents both on road and on farm is being conducted by Professor B.T. Stephanson, Associate Professor, Department of Agricultural Engineering, University of Alberta. The frequent occurrence of fatalities arising from road accidents prompted the research leading to this thesis.

Year	Total Deaths	Percent Associated With Roll Over	Sideways (%)	Rear (%)	Front (%)	Direction Not Known (%)
1963	12	83	66.5	16.5	0	0
1964	12	92	92	0	0	0
1965	9	66.6	44.5	22	0	0
Total	33	82	70	12	0	0

Road Accidents

Year	Total Deaths	Percent Associated With Roll Over	Sideways (%)	Rear (%)	Front (%)	Direction Not Known (%)
1963	13	31	7.7	23.3	0	0
1964	17	35	23.5	5.75	0	5.75
1965	6	50	33.3	16.7	0	0
Total	36	36	19.5	14.0	0	2.5

Farm Accidents

Table 1. A Preliminary Analysis of Data Obtained by Stephanson.

It has been reported[1] that among 686 tractor accidents examined in Sweden during the period from 1954 to 1958, 121 or 18% referred to sideways overturning and 52 or 8% to overturning backwards. In the first case 54 people lost their lives and in the second 20. The other tractor accidents during the period led to 22 fatalities. The largest number of people killed during one year was 19 as a result of sideways overturning and 4 by rearing.

Of the serious accidents, the majority took place in hilly country and resulted in the tractor overturning sideways, due to the unevenness or slope of the ground, the tractor being overthrown by an insufficiently braked trailer, or by an overturning trailer.

According to J.R. Whitaker[10] one agricultural worker in 20 had an accident every year. Tractors were involved in nearly 30% of fatal accidents and about 8% of non-fatal ones. They are the biggest individual cause of deaths and could be considered the most important problem in accident prevention.

From the previous statistics it is clear that sideways overturning is one of the major types of tractor accidents. More work has been done on longtitudinal, or pitch stability of tractors than on roll stability. The purpose of this project was the systematic investigation of the stability of a tractor and to see the effect of different combinations of existing conditions. Use of an analog computer made it possible to investigate unsafe areas of operation.

1.2 OBJECTIVES

The objectives of this thesis were as follows:-

1. The simulation of equations representing dynamic behaviour of a farm tractor on an analog computer.
2. To see the effect of different types of ballasts, namely 75% liquid filled and lead ballast, and to compare the effects with 100% air filled tires on the stability of a tractor.
3. To see the effect of the turning radius on the stability of a tractor running over random bumps.
4. To see the effect of speed on the stability of a tractor running over random bumps.
5. To see the effect of slope on the stability of a tractor running over random bumps.

2. REVIEW OF LITERATURE

According to Gough[8] forty tractor drivers were killed in 1959 because their tractors overturned. "Many of them were drivers with experience so it should not be assumed that only new drivers suffer this type of accident."

All previous statistics stress the need for more research work on the dynamic behaviour of farm tractors and safety measures for preventing accidents.

Sauve and McKibben[16] found that when rubber tired tractors were subjected to shocks, magnitude of acceleration was greatest for the 100% liquid filled tires and least for the 90% liquid filled. "There was little difference in the total number of shocks in 100% air filled and 100% liquid filled tires. In all cases the period of vibration was decreased and the rate of damping was increased by increasing the extent of liquid fill. On the other hand the rate of axle rise or vertical acceleration was increased by increasing the percent of liquid. When crossing a simple obstruction the height of rise was increased as the percent of liquid fill was increased. However in dropping from a higher to a lower level or in crossing a depression the situation was reversed."

Dahir and Stout[5] studied the effects of liquid and dry ballasts on the stability of pneumatic tired tractors. Their study was divided into two parts.

1. The tractor was suspended in the air and the dynamic effects of three types of ballast were measured.
2. The tractor was driven over different arrangements of five inch bumps to determine the resultant vertical forces on the axle, the effects of bump spacing on the resultant forces and the shock absorbing characteristics of different types of ballast.

They concluded the following.

1. Tractor suspended in air.

   a. Each liquid fill exerted a different force on the axle housing at speeds above 120 rpm. The 90% fill exhibited the maximum force.

   b. Unstabilizing forces exerted by liquid ballast in the tractor tires appeared to be insufficient to be important factors in contributing to tractor upsets.

   c. Dry ballast in the tires resulted in upward forces ranging from 51 to 550 lbs. at speeds from 58 to 162 rpm. (8.9 to 25 mph)

2. Tractor running over bumps.

   a. Under the conditions of this test the vertical resultant forces on the axle housing were greater with liquid fill than with dry ballast or cast iron wheel weights.

   b. Upward forces on the axle housing of the order of 5,000 to 10,000 lb. resulted at 74 rpm (11.4 mph) with a bump spacing of 6.5 feet for the 100% liquid filled ballast tests.

   c. The upward forces on the axle housing at speeds from 30 to 74 rpm (4.6 to 11.4 mph) and bump spacing of 6.5 feet were as follows:

      i. Ninety percent liquid filled-2,700 to 4,500 lb.

      ii. Seventy-five percent liquid filled-3,000 to 4,900 lb.

      iii. Dry ballast fill-2,200 to 3,500 lb.

      iv. Cast iron wheel weights 1,000 to 3,700 lb.

   d. Larger forces were observed at a bump spacing of 6.5 feet than at a spacing of 13 feet.

e. Speeds as low as 30 rpm (4.6 mph) coupled with "unfavourable" conditions might cause tractor upsets.

f. Forces of magnitude high enough to cause tractor upsets under "unfavourable" conditions were encountered with all types of ballasts tested.

The highest forces occurred at 74 rpm (11.5 mph) with two pairs of bumps 6.5 feet apart. Runs beyond this speed were not made because the severity of the shocks was so great that higher speeds were considered unsafe.

In general at higher speeds, the resultant vertical forces tended to decrease. This phenomenon could be due to the moment of the tractor. Movies taken during the tests revealed that the tractor hit the bumps and slid forward instead of bouncing vertically as it did at lower speeds.

H.W. Sack[15] gave the equation

$\tan \beta = (Lcg - \mu H)/(Hcg - H)$ for evaluating the stability of any tractor.

where

β = angle of gradient (degree)

Lcg = distance of centre of gravity ahead of rear axle centre line (inches)

μ = traction coefficients (dimensionless)

H = height of drawbar above ground level (inches)

Hcg = height of centre of gravity above ground level (inches)

According to Sack[15] by means of this equation the longitudinal stability performance of any existing or proposed tractor design can be evaluated more readily.

Experiments conducted by Knapp[11] with a plastic prototype tractor tire revealed that there was a critical speed at which the instability of the fluid in a 3/4 filled tire causes definite dynamic inbalance. Calculations projected from these tests to wheels of actual size yielded forces from 1,024 lb upward to 1,536 lb downward. The percentage of fill and viscosity of the liquid were also contributing factors.

According to Worthington[21] on hard level ground conditions where wheel traction was greatest, the tire reaction with the ground was experimentally found to be located directly below the rear axle centre line and at the surface of the ground. "Under soft soil and sandy conditions, the drive wheel sinks into the soil, increasing the rolling resistance and decreasing available draw-bar pull. Under such conditions the point of drive wheel reaction with the soil probably exists as a virtual centre, but in view of the low drawbar pull evident at such times, it must be concluded that the most unfavourable overall conditions for dynamic stability are probably present when operating on hard ground."

According to an informal working bulletin[1] sideways overturning was the most common form of serious accidents. "A tractor's high centre of gravity, short wheel base and individual brakes, enabling very short turns to be executed make a tractor relatively easy to overturn. If the centre of gravity is raised and shifted forwards, as a result of having a front loader and a filled bucket in the raised position distance C in Figure 1 may easily be reduced to zero if one rear wheel passes over a bump or drops into a hollow.

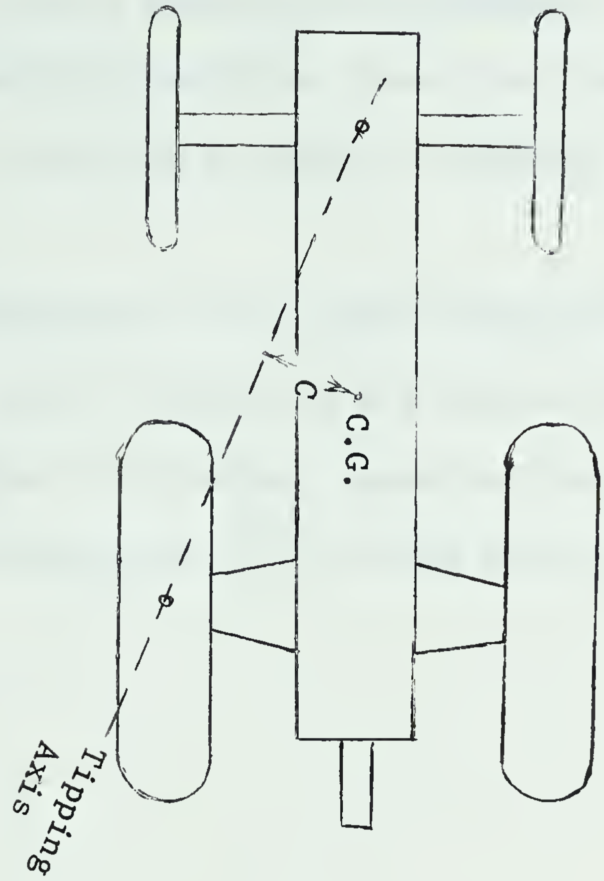


Figure 1: The Moment Arm of Gravity in Relation to Tipping Axis.

The net result is that the stability of a tractor against sideways tipping may be less than is imagined and that the need for a wide wheel spacing at the rear is emphasized.

The higher the centre of gravity the greater the tipping moment brought about by the centrifugal force, while the greater the wheel track of the rear wheels, the greater the restraining moment exerted by the force of gravity. Thus from this aspect it is of advantage to have a low centre of gravity and a wide wheel track. If a curve is incorrectly banked or if a turn is made on a slope with the wheels on the outer radius further down the slope than the wheels on the inner radius, the risk of tipping is increased. The risk is also greater if the ground is uneven , so that one wheel comes higher than the other as the tractor turns."

Raney[13] (in 1959) represented the dynamic behaviour of farm tractors mathematically and then showed that these equations could be solved with a sufficient degree of accuracy by using an electronic analog computer.

Delzell[6] found that in all cases when hitting the obstructions, the bounce of the air filled tire was greater than that of the liquid filled tire. "In low gear operation the tractor did not bounce as high as the obstruction, but in high gear it bounced beyond the obstruction."

3. THEORY

3.1 TRACTOR SUSPENSION SYSTEMS

Rubber tires behave as springs. Since wheeled tractors are normally mounted on rubber tires they can be treated as a mass suspended on springs. Tractors have six possible degrees of freedom in which vibrations can occur. When the tractor wheels are locked it can vibrate in three principal directions (x,y, and z) as shown in Figure 2 and it can rotate about any one of its three principal axes (longitudinal, transverse or vertical Φ,Θ, or β) as shown in Figure 2. In other words, it can roll, pitch and yaw.

In this project only three degrees of freedom were considered.

1. Vertical motion of chassis.
2. Rotation about the transverse axis.
3. Rotation about the longitudinal axis..

The other possible degrees of freedom were not considered because it was felt that their overall contribution to the stability problem were small.

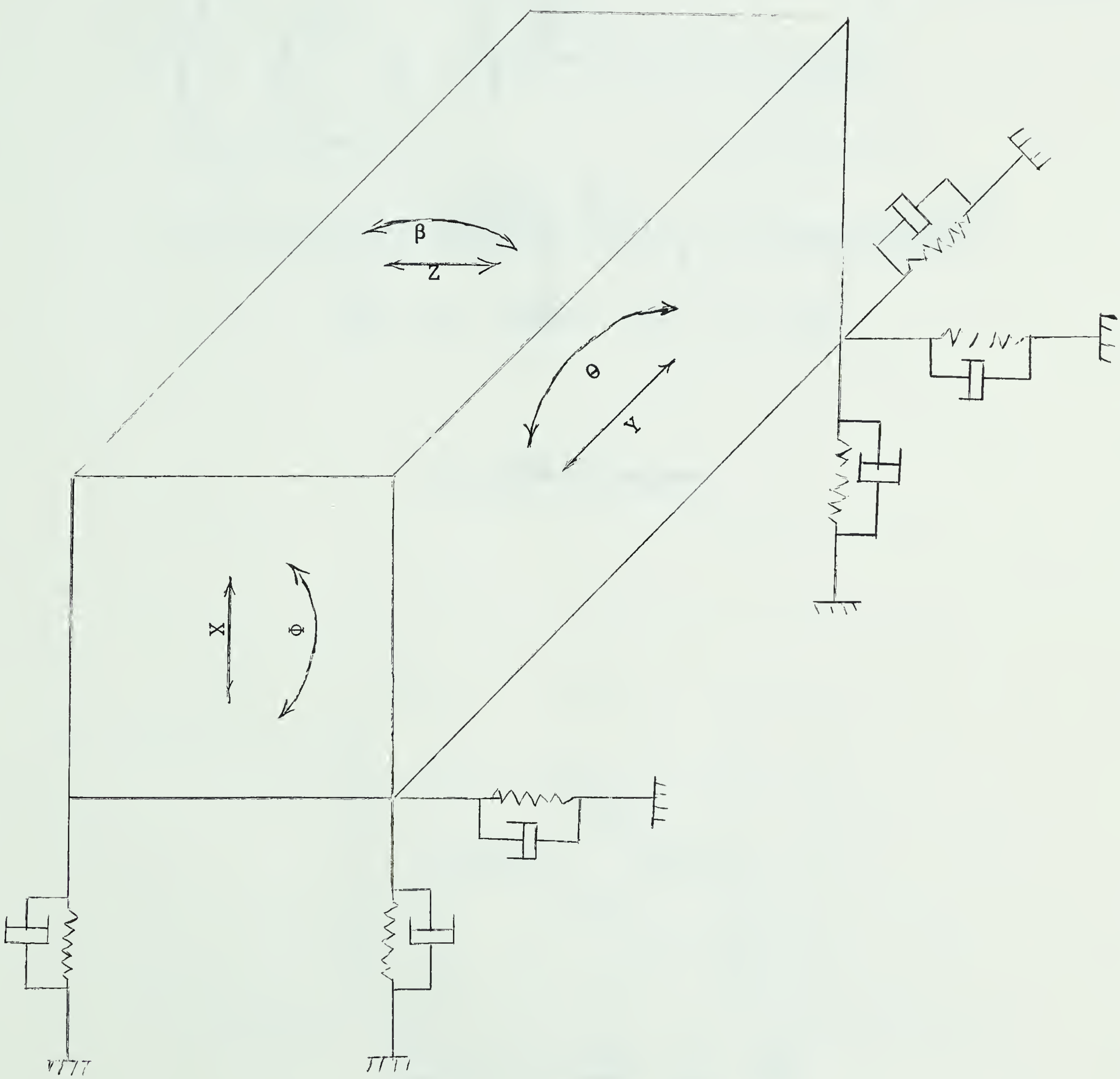
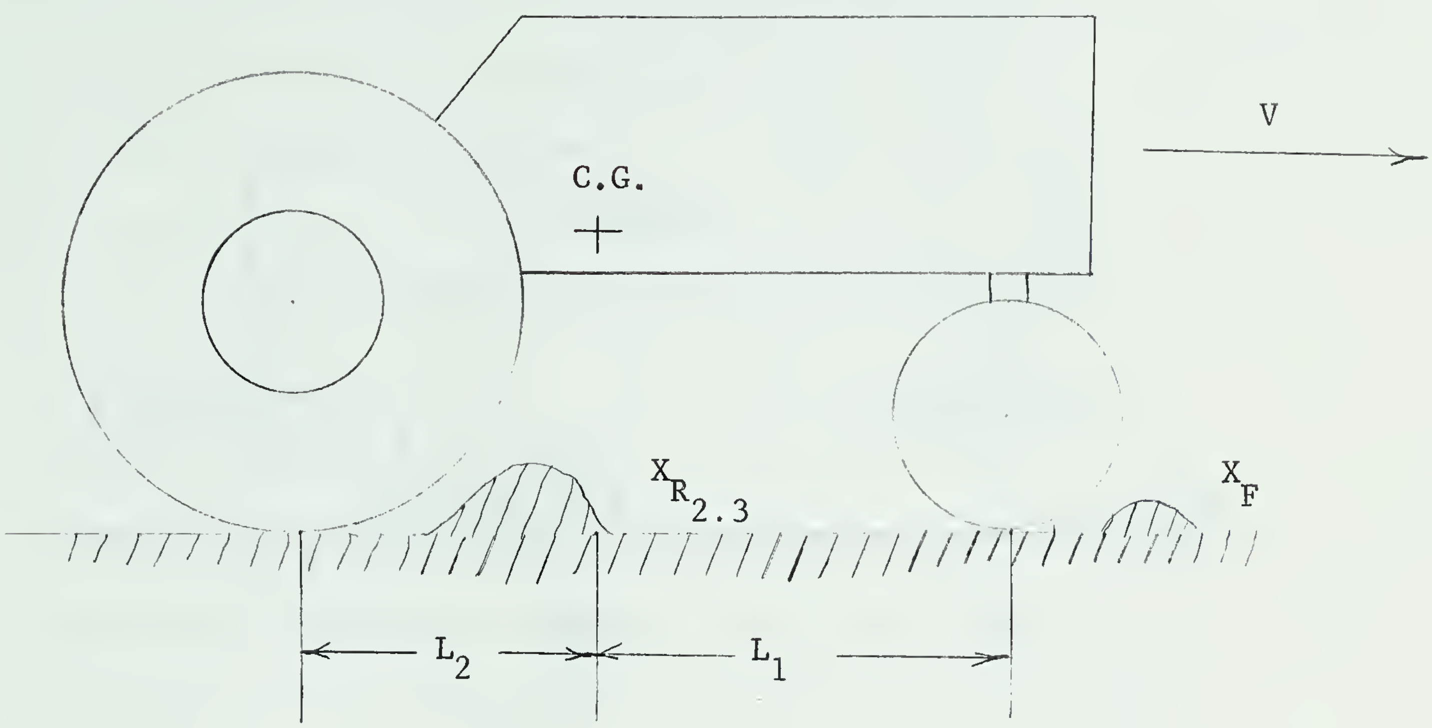
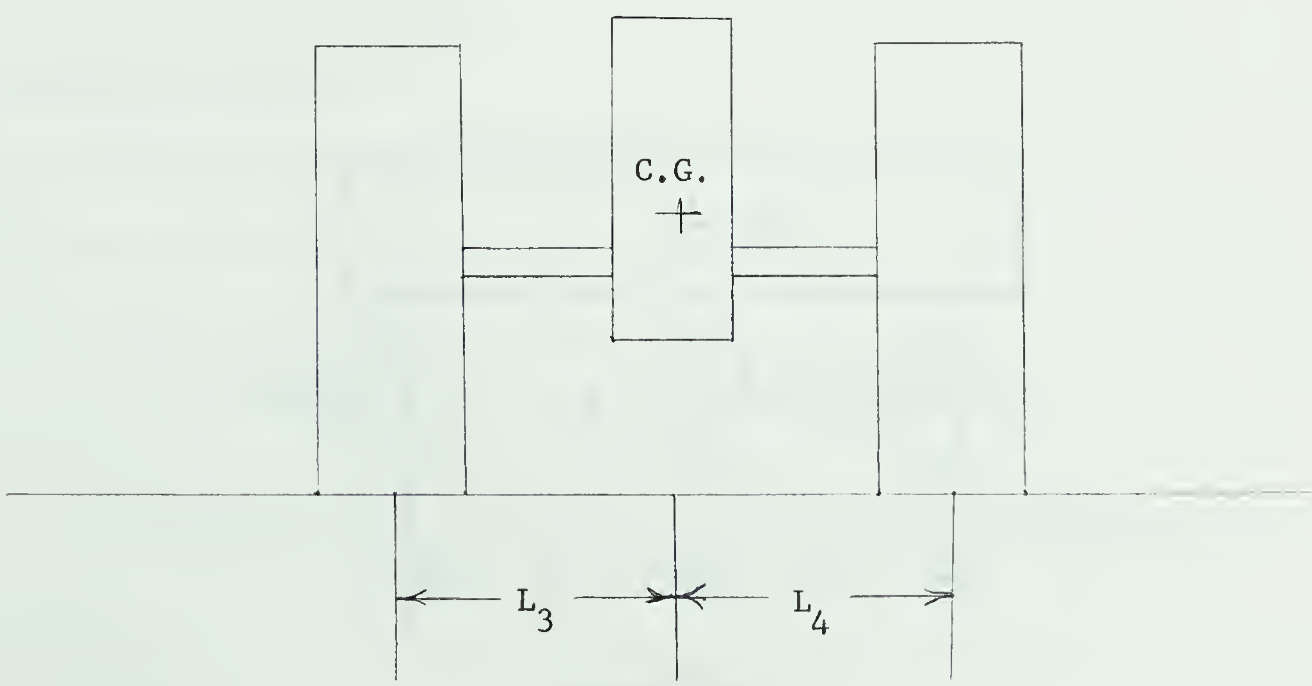


Figure 2: Tractor as a Spring Mass System with Six Possible Degrees of Freedom.

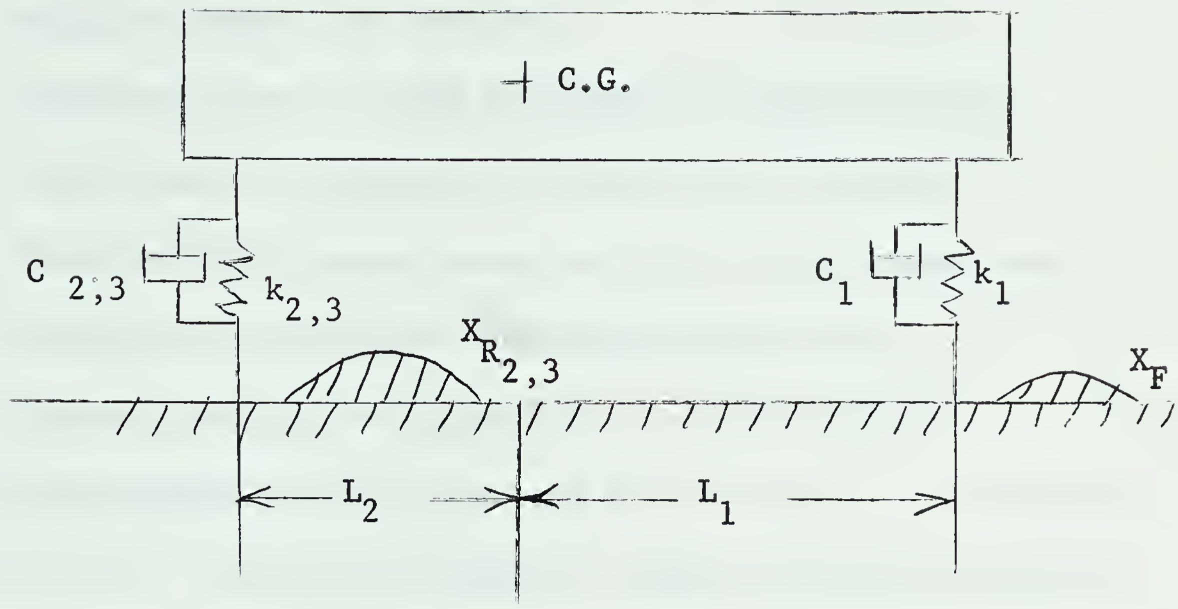


SIDE VIEW OF TRACTOR

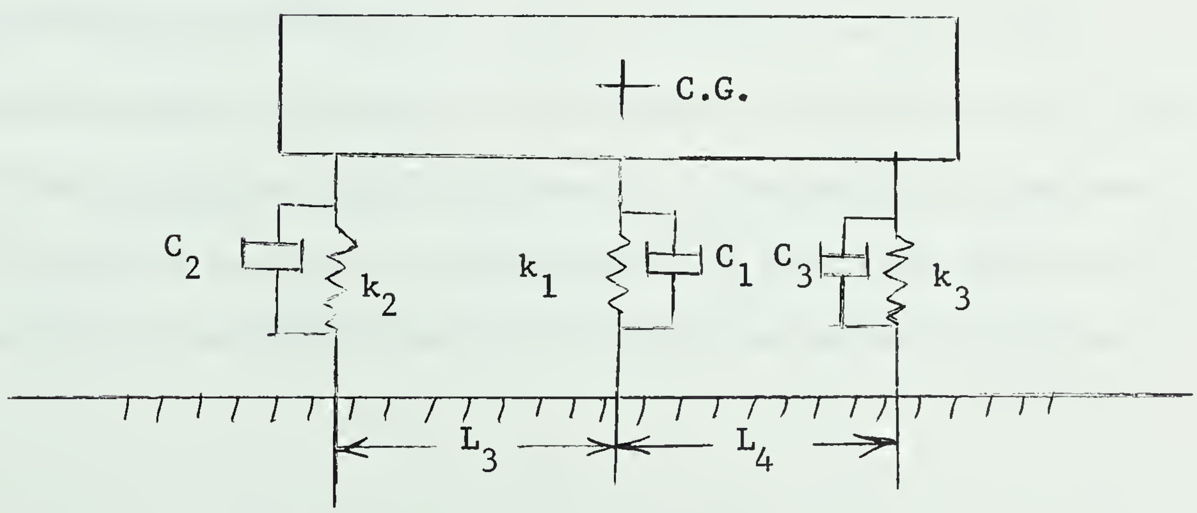


REAR VIEW OF TRACTOR

Figure 3: Principal Dimensions of a Tractor.



SIDE VIEW



REAR VIEW

Figure 4: Schematic Representation of Tractor System.

3.2 ASSUMPTIONS MADE IN SETTING UP THE MATHEMATICAL MODEL

1. Tractor tires follow Hooke's Law and have a surface contact with the supporting surface.
2. The tractor was a rigid body made of frame and axle.
3. Only bumps to 6 inches in height were considered.
4. When the tire leaves the ground there is no restoring spring force acting at that particular tire.
5. Viscous damping was assumed in the tires. Raney[13].
6. Displacement of the following three points was considered.
   - i. Vertical movement of centre of the front axle.
   - ii. Vertical movement of the centre of the left rear wheel.
   - iii. Vertical movement of the centre of the right rear wheel.
7. There is enough resistance between soil and tire to prevent any lateral slippage.

3.3 MATHEMATICAL MODEL

In developing the mathematical model, movement of the three points were considered. They were the centres of the front axle, left rear wheel and right rear wheel. Movement of the centre of the front axle was considered because any displacement in the front wheels will be transferred to the pivot point which is at the center of the axle.

3.3.1 Coordinate System

The following coordinate system was used throughout the thesis.

Clockwise ↑	↓
+ Angular	+ Linear
Displacement	Displacement
Velocity	Velocity
Acceleration	Acceleration
+ Torque	+ Force

Schematic representation of the geometry of the system is shown in Figure 5.

3.3.2 Symbols and Notations

The following symbols and notations were used in deriving the system equations.

L_1 = Distance from the centre of gravity to the front axle.

L_2 = Distance from the centre of gravity to the rear axle.

L_3 = Distance from the centre of gravity to the centre of the left rear wheel.

L_4 = Distance from the centre of gravity to the centre of the right rear wheel.

X_F = Disturbance seen by the front tires.

$X_{R_{2-3}}$ = Disturbance seen by the left and right rear wheels.

X_1 = Vertical displacement of the centre of the front axle.

X_2 = Vertical displacement of the left rear wheel axle centre.

X_3 = Vertical displacement of the right rear wheel axle centre.

X_4 = Vertical displacement of the centre of the rear wheels axle.

X = Vertical displacement of the centre of gravity.

F = Centrifugal force due to turning.

m = Mass of tractor.

$I\Theta$ = Mass moment of inertia about a transverse axis through the centre of gravity.

$I\Phi$ = Mass moment of inertia about a longitudinal axis through the centre of gravity.

Θ = Pitch about the centre of gravity.

Φ = Roll about the centre of gravity.

k_1 = Spring rate of the front tires taken together.

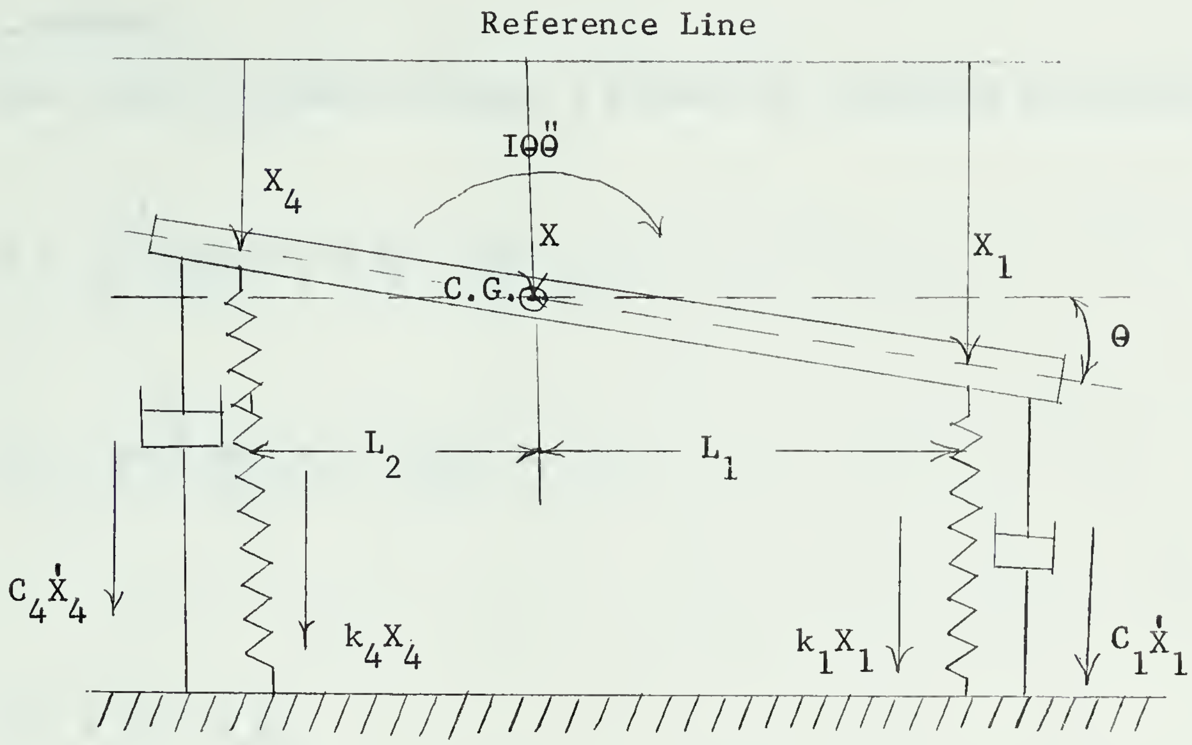
k_2 = Spring rate of the left rear tire.

k_3 = Spring rate of the right rear tire.

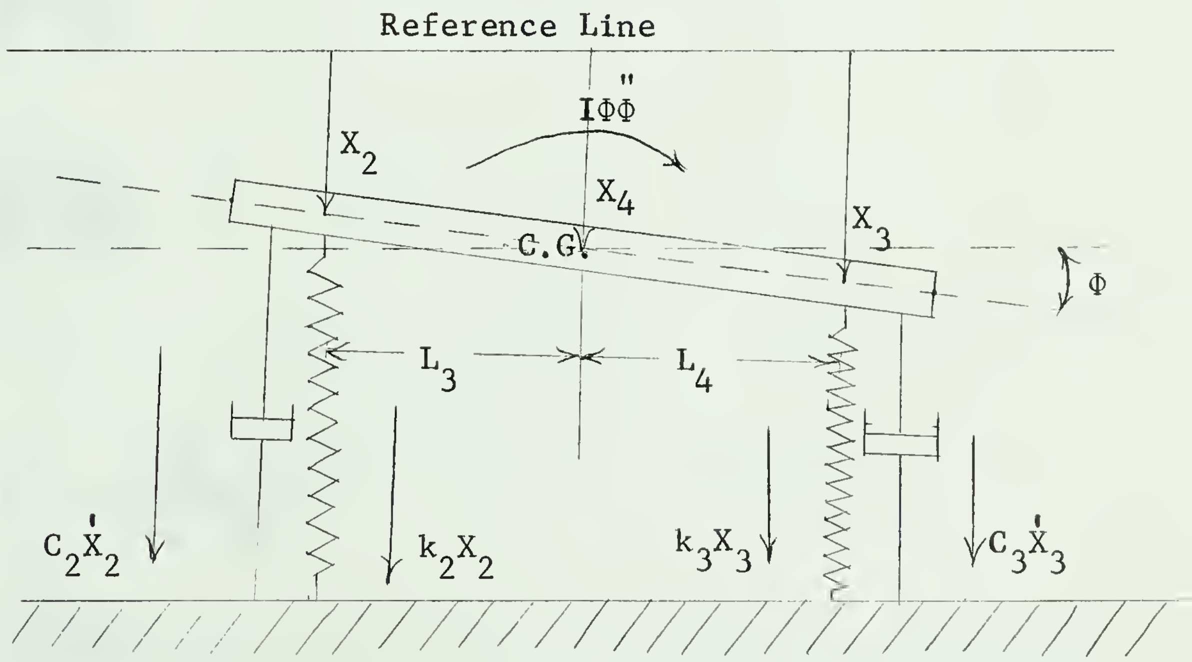
C_1 = Viscous damping constant for both front tires.

C_2 = Damping constant for left rear tire.

C_3 = Damping constant for right rear tire.



SIDE VIEW



REAR VIEW

Figure 5: Schematic Representation of Geometry of System.

3.3.3 Geometry

The geometry shown in Figure 5 yields the following relationship.

$$X = \frac{L_1}{L_1 + L_2}(X_4 - X_1) + X_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$X_4 = \frac{L_4}{L_3 + L_4}(X_2 - X_3) + X_3 \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Or

$$X = K_1X_1 + K_2X_4$$

$$X_4 = K_3X_2 + K_4X_3$$

$$\Theta = \frac{X_1 - X_4}{L_1 + L_2}$$

$$\Phi = \frac{X_3 - X_2}{L_3 + L_4}$$

Where

$$K_1 = \left(1 - \frac{L_1}{L_1 + L_2} \right)$$

$$K_2 = \frac{L_1}{L_1 + L_2}$$

$$K_3 = \frac{L_4}{L_3 + L_4}$$

$$K_4 = \left(1 - \frac{L_4}{L_3 + L_4} \right)$$

for the symmetrical case

$$K_3 = K_4 = 0.5$$

Referring to Figure 5 (top) and summing forces yields,

$$\Sigma F_X = 0$$

$$m \frac{d^2X}{dt^2} + C_1 \frac{dX_1}{dt} + C_2 \frac{dX_2}{dt} + C_3 \frac{dX_3}{dt} + k_1X_1 + k_2X_2 + k_3X_3 = 0 \quad \ldots\ldots (3)$$

Referring again to Figure 5 (top) and summing torque yields,

$$\Sigma T\Theta = 0$$

$$I\Theta \frac{d^2\Theta}{dt^2} + L_1C_1 \frac{dX_1}{dt} - L_2C_2 \frac{dX_2}{dt} - L_2C_3 \frac{dX_3}{dt} +$$

$$L_1k_1X_1 - L_2k_2X_2 - L_2k_3X_3 = 0 \quad \ldots\ldots (4)$$

Referring to Figure 5 (bottom) and summing torque yields,

$$\Sigma T\Phi = 0$$

$$I\Phi \frac{d^2\Phi}{dt^2} + L_4C_3 \frac{dX_3}{dt} - L_3C_2 \frac{dX_2}{dt} + L_4k_3X_3 - L_3k_2X_2 = 0 \quad \ldots\ldots (5)$$

Considering the geometric relationships as established in Equations 1 and 2 and substituting in Equation 3 yields,

$$m\ddot{X} + C_1\dot{X}_1 + C_2\dot{X}_2 + C_3\dot{X}_3 + k_1X_1 + k_2X_2 + k_3X_3 = 0$$

In this thesis X' and X'' are the equivalent of $\dot{X}$ and $\ddot{X}$ respectively.

$$\ddot{X} = -\frac{C_1}{m}\dot{X}_1 - \frac{C_2}{m}\dot{X}_2 - \frac{C_3}{m}\dot{X}_3 - \frac{k_1}{m}X_1 - \frac{k_2}{m}X_2 - \frac{k_3}{m}X_3$$

$$K_1\ddot{X}_1 + K_2\ddot{X}_4 = -\frac{C_1}{m}\dot{X}_1 - \frac{C_2}{m}\dot{X}_2 - \frac{C_3}{m}\dot{X}_3 - \frac{k_1}{m}X_1 - \frac{k_2}{m}X_2 - \frac{k_3}{m}X_3$$

$$\ddot{X}_1 = -\frac{C_1}{K_1 m}\dot{X}_1 - \frac{C_2}{K_1 m}\dot{X}_2 - \frac{C_3}{K_1 m}\dot{X}_3 - \frac{k_1}{K_1 m}X_1 - \frac{k_2}{K_1 m}X_2 - \frac{k_3}{K_1 m}X_3 - K_2(K_3\ddot{X}_2 + K_4\ddot{X}_3)$$

$$\ddot{X}_1 = -\frac{C_1}{K_1 m}\dot{X}_1 - \frac{C_2}{K_1 m}\dot{X}_2 - \frac{C_3}{K_1 m}\dot{X}_3 - \frac{k_1}{K_1 m}X_1 - \frac{k_2}{K_1 m}X_2 - \frac{k_3}{K_1 m}X_3 - \frac{K_2K_3}{K_1}(\ddot{X}_2 + \ddot{X}_3)$$

$$\ddot{X}_1 = -K_5\dot{X}_1 - K_6(\dot{X}_2 + \dot{X}_3) - K_7X_1 - K_8(X_2 + X_3) - K_9(\ddot{X}_2 + \ddot{X}_3)$$

Where

$$K_5 = \frac{C_1}{mK_1}$$

$$K_6 = \frac{C_2}{mK_1}$$

$$K_7 = \frac{k_1}{mK_1}$$

$$K_8 = \frac{k_2}{mK_1}$$

$$K_9 = \frac{K_2K_3}{K_1}$$

Substituting proper geometric relationships in Equation 4 yields,

$$I\theta \frac{d^2\theta}{dt^2} + L_1C_1 \frac{dX_1}{dt} - L_2C_2 \frac{dX_2}{dt} - L_2C_3 \frac{dX_3}{dt} + L_1k_1X_1 - L_2k_2X_2 - L_2k_3X_3 = 0$$

$$I\theta \ddot{\theta} + L_1C_1\dot{X}_1 - L_2C_2(\dot{X}_2 + \dot{X}_3) + L_1k_1X_1 - L_2k_2(X_2 + X_3) = 0$$

$$I\theta \ddot{\theta} = - L_1C_1\dot{X}_1 + L_2C_2(\dot{X}_2 + \dot{X}_3) - L_1k_1X_1 + L_2k_2(X_2 + X_3)$$

$$\ddot{\theta} = - \frac{L_1C_1\dot{X}_1}{I\theta} + \frac{L_2C_2}{I\theta} (\dot{X}_2 + \dot{X}_3) - \frac{L_1k_1}{I\theta} X_1 + \frac{L_2k_2}{I\theta} (X_2 + X_3)$$

$$\frac{1}{L_1 + L_2} (\ddot{X}_1 - \ddot{X}_4) = - \frac{L_1C_1}{I\theta} \dot{X}_1 + \frac{L_2C_2}{I\theta} (\dot{X}_2 + \dot{X}_3) - \frac{L_1k_1}{I\theta} X_1 + \frac{L_2k_2}{I\theta} (X_2 + X_3)$$

$$\ddot{X}_1 - (K_3\ddot{X}_2 + K_4\ddot{X}_3) = (L_1 + L_2) \left\{ -\frac{L_1C_1}{I\theta} \dot{X}_1 + \frac{L_2C_2}{I\theta} (\dot{X}_2 + \dot{X}_3) - \frac{L_1k_1}{I\theta} X_1 + \frac{L_2k_2}{I\theta} (X_2 + X_3) \right\}$$

$$-K_3\ddot{X}_2 = K_4\ddot{X}_3 - \ddot{X}_1 + (L_1 + L_2) \left\{ - \frac{L_1C_1}{I\theta} \dot{X}_1 + \frac{L_2C_2}{I\theta} (\dot{X}_2 + \dot{X}_3) - \frac{L_1k_1}{I\theta} X_1 + \frac{L_2k_2}{I\theta} (X_2 + X_3) \right\}$$

$$-\ddot{X}_2 = \frac{K_4}{K_3} \ddot{X}_3 - \frac{\ddot{X}_1}{K_3} - (L_1 + L_2) \frac{L_1C_1}{I\theta K_3} \dot{X}_1 + (L_1 + L_2) \frac{L_2C_2}{I\theta K_3} (\dot{X}_2 + \dot{X}_3) - (L_1 + L_2) \frac{L_1k_1}{I\theta k_3} X_1 + (L_1 + L_2) \frac{L_2k_2}{I\theta K_3} (X_2 + X_3)$$

$$\ddot{X}_2 = - \ddot{X}_3 + 2\ddot{X}_1 + (L_1 + L_2) \frac{L_1C_1}{I\Theta k_3} \dot{X}_1 - (L_1 + L_2) \frac{L_2C_2}{I\Theta K_3} (\dot{X}_2 + \dot{X}_3) +$$

$$(L_1 + L_2) \frac{L_1k_1}{I\Theta K_3} X_1 - (L_1 + L_2) \frac{L_2k_2}{I\Theta k_3} (X_2 + X_3)$$

$$\ddot{X}_2 = - K_{10}\ddot{X}_3 + K_{11}\ddot{X}_1 + K_{12}\dot{X}_1 - K_{13}(\dot{X}_2 + \dot{X}_3) + K_{14}X_1 - K_{15}(X_2 + X_3)$$

Where

$$K_{10} = 1$$

$$K_{11} = 2$$

$$K_{12} = (L_1 + L_2) \frac{L_1C_1}{I\Theta K_3}$$

$$K_{13} = (L_1 + L_2) \frac{L_2C_2}{I\Theta K_3}$$

$$K_{14} = (L_1 + L_2) \frac{L_1k_1}{I\Theta K_3}$$

$$K_{15} = (L_1 + L_2) \frac{L_2k_2}{I\Theta K_3}$$

Substituting proper geometric relations in Equation 5 yields,

$$I\Phi \frac{d^2\Phi}{dt^2} + L_4C_3 \frac{dX_3}{dt} - L_3C_2 \frac{dX_2}{dt} + L_4k_3X_3 - L_3k_2X_2 = 0$$

$$I\Phi\,\ddot{\Phi} + L_4 C_3 \dot{X}_3 - L_3 C_2 \dot{X}_2 + L_4 k_3 X_3 - L_3 k_2 X_2 = 0$$

$$I\Phi\,\ddot{\Phi} = - L_4 C_3 (\dot{X}_3 - \dot{X}_2) - L_4 k_3 (X_3 - X_2)$$

$$\ddot{\Phi} = - \frac{L_4 C_3}{I\Phi} (\dot{X}_3 - \dot{X}_2) - \frac{L_4 k_3}{I\Phi} (X_3 - X_2)$$

$$\frac{1}{L_3 + L_4} (\ddot{X}_3 - \ddot{X}_2) = - \frac{L_4}{I\Phi} C_3 (\dot{X}_3 - \dot{X}_2) - \frac{L_4 k_3}{I\Phi} (X_3 - X_2)$$

$$\ddot{X}_3 = - \frac{L_4 C_3}{I\Phi} (L_3 + L_4) (\dot{X}_3 - \dot{X}_2) - \frac{L_4 k_3}{I\Phi} (L_3 + L_4) (X_3 - X_2) + \ddot{X}_2$$

$$\ddot{X}_3 = K_{16} \ddot{X}_2 - K_{17} (\dot{X}_3 - \dot{X}_2) - K_{18} (X_3 - X_2)$$

Where

$$K_{16} = 1$$

$$K_{17} = \frac{L_4 C_3}{I\Phi} (L_3 + L_4)$$

$$K_{18} = \frac{L_4 k_3}{I\Phi} (L_3 + L_4)$$

4. PROCEDURE FOR DETERMINING THE TRACTOR AND TIRE CONSTANTS

A Massey Ferguson 135 gasoline tractor was used for the test. It was mounted on 13.6 x 24, 4 ply tires. The same tires were used for the 75% liquid filled ballast case. 13.6 x 28, 4 ply tires were used for testing with lead ballast.

For simulating the system equations on the computer, knowledge of the coefficients involved in the equations was very essential. A tractor was used for finding out these coefficients. The whole study was made for the particular tractor used but the fundamentals apply to all tractors.

4.1 ECCENTRIC ROLLER FOR THE TRACTOR

Two systems were employed to give sinusoidal input to the system. In one, peak to peak sinusoidal input to the system was three inches. This was achieved by mounting a seven foot long eleven inch diameter pipe on bearings with one and a half inches of eccentricity. In the other, a platform was driven by an eccentric roller, which was driven by tractor pto.

In an attempt to simulate field behaviour the tractor was held from moving forward on the roller (Figure 6) but was free to move vertically. The roller was driven by the tractor's own power. When the tractor was running, roller speed was recorded and at the same time, a series of movie films of the center of the rear axle were taken. The tractor was run at different speeds and a series of movies were taken. It was not possible to run the roller at a speed of more than 100 rpm when the tractor was on the roller.

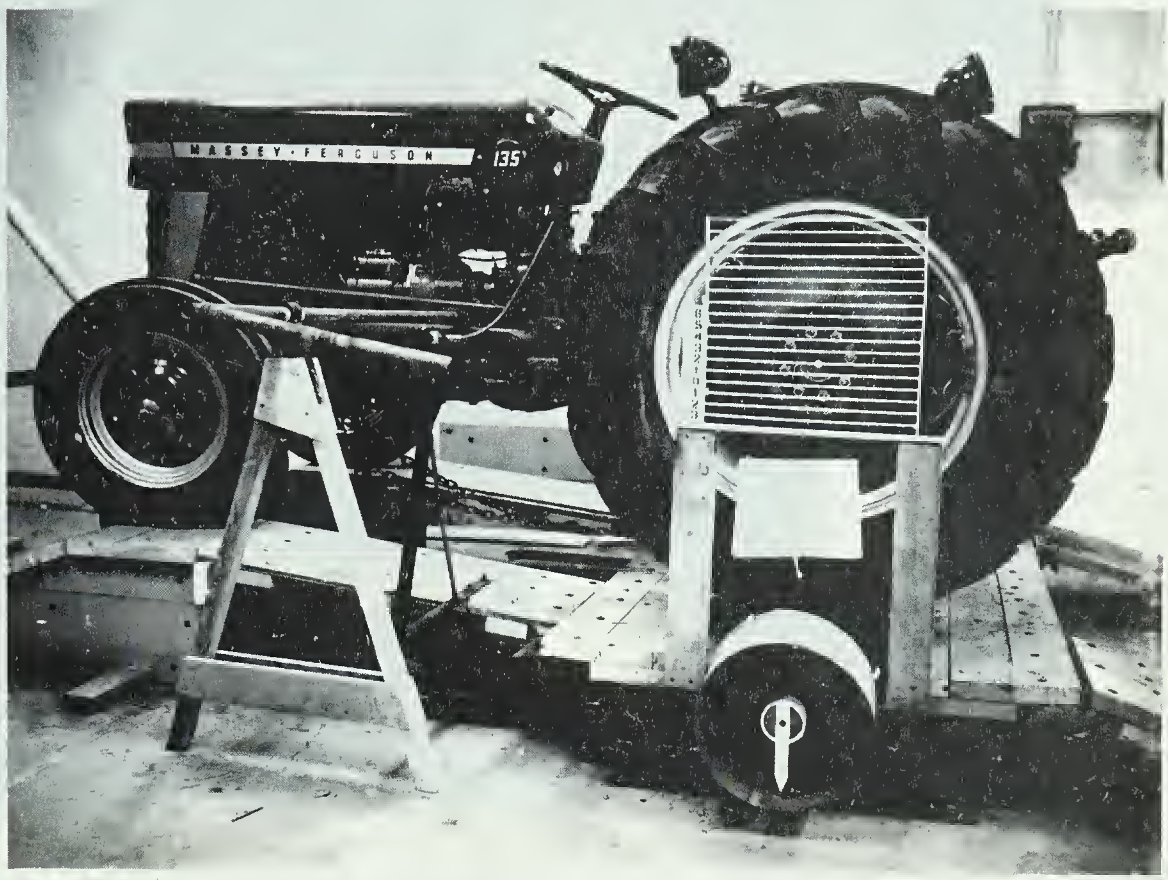


Figure 6: Tractor on Eccentric Roller (System No. 1)

System No. 2 (Figure 7) was used to provide sinusoidal input in a great range of speeds, making it possible to obtain the natural frequency. At the same time it was also used to see the difference in behaviour under the two systems of input.

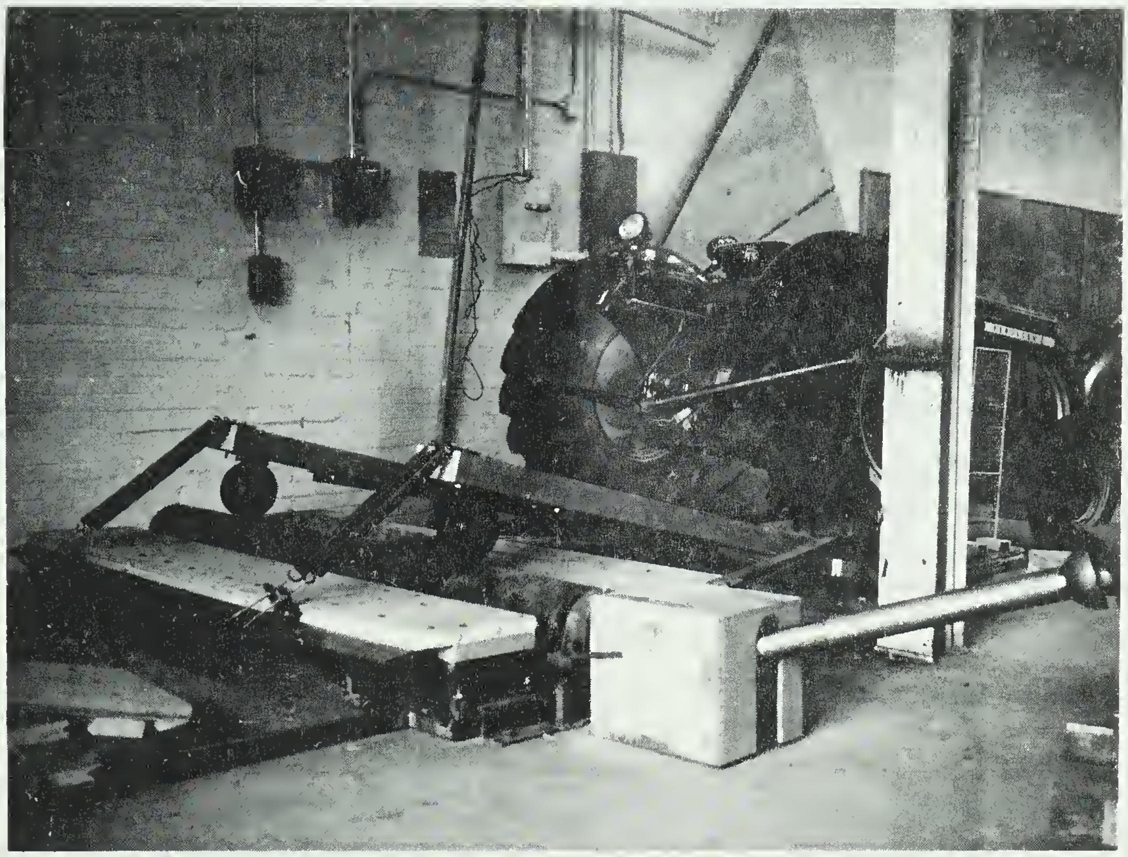


Figure 7: Tractor on System No. 2.

4.2 POINTER AND GRID

A grid was constructed and used for measuring output amplitudes of the tractor tire. A pointer was fixed in the shaft to give the phase angle at a particular time.

4.3 CAMERA

A camera was used to take movie films when the tractor was under test. Films were taken at a speed of 23 frames per second. These films were used in determining output amplitudes.

4.4 AMPLITUDE RATIO VERSUS FREQUENCY CURVE FOR THE TIRES WITH AND WITHOUT BALLAST.

Results of an analysis of the movies taken when the tractor was on the eccentric roller (Figure 6) are tabulated below.

Table 2: Tractor on 100% Air Filled Tire. Air Pressure 12 psig

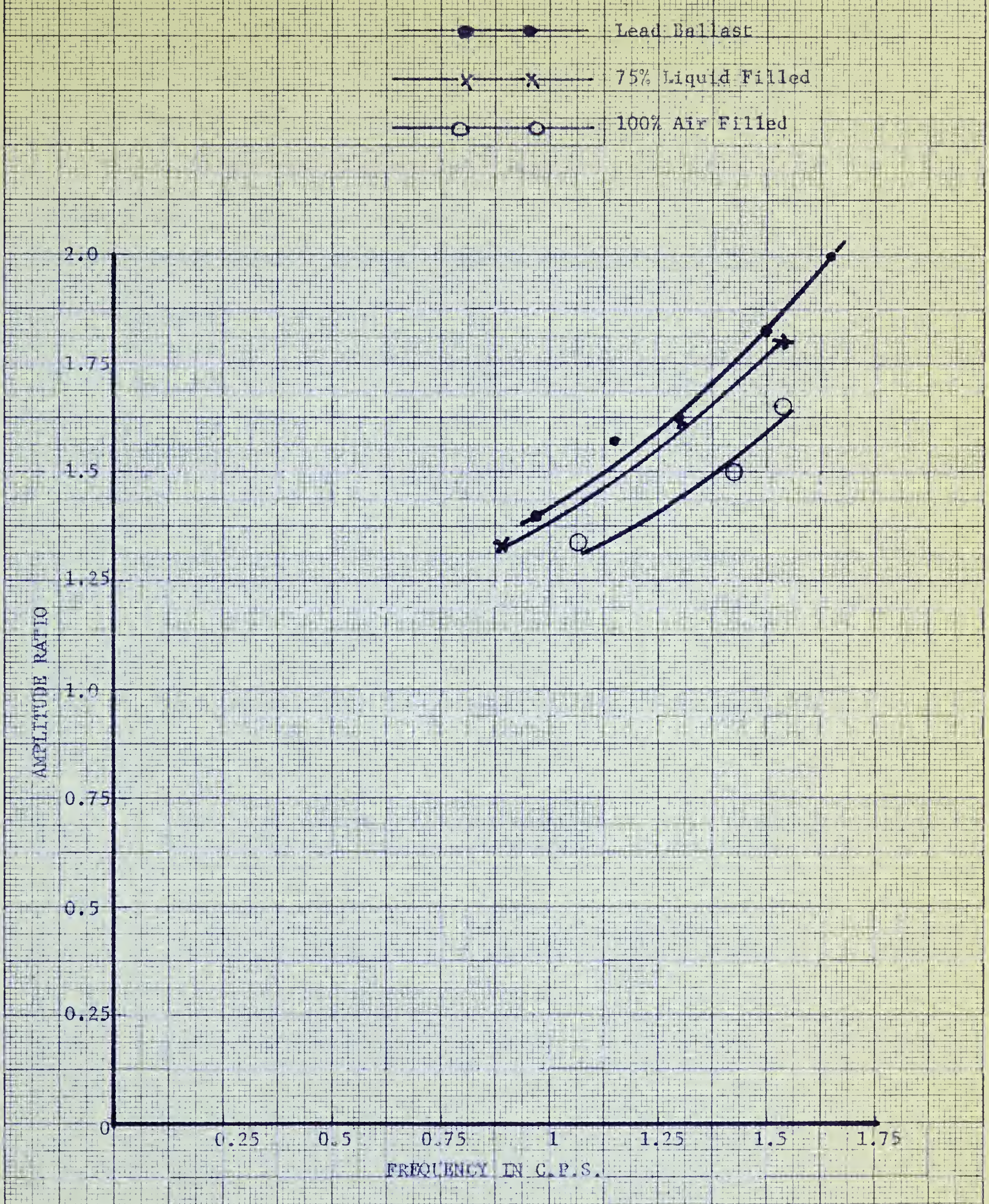
S. No.	Speed in rpm	Frequency in cps	Peak to Peak input amplitude in inches	Peak to Peak output amplitude in inches	Amplitude ratio
1	64	1.1	3.0	4.0	1.3
2	86	1.4	3.0	4.5	1.5
3	92	1.5	3.0	5.0	1.7

Table 3: Tractor on 75% Liquid Filled Tire. Air Pressure 12 psig

S. No.	Speed in rpm	Frequency in cps	Peak to Peak input amplitude in inches	Peak to Peak output amplitude in inches	Amplitude ratio
1	53	0.88	3.0	4.0	1.3
2	78	1.3	3.0	4.9	1.6
3	92	1.5	3.0	5.5	1.8

Table 4: Tractor on lead ballast tires.

S. No.	Speed in rpm	Frequency in cps	Peak to Peak input amplitude in inches	Peak to Peak output amplitude in inches	Amplitude ratio
1	58	0.97	3.0	4.2	1.4
2	70	1.1	3.0	4.7	1.5
3	90	1.5	3.0	5.5	1.8
4	100	1.7	3.0	6.0	2.0



Graph 1: Comparison of Three Outputs on Eccentric Roller.
(System No. 1)

System No. 2 was used to give sinusoidal input at higher speeds. For checking non linearity of spring and damping coefficients of the system two different inputs were used for the front tires.

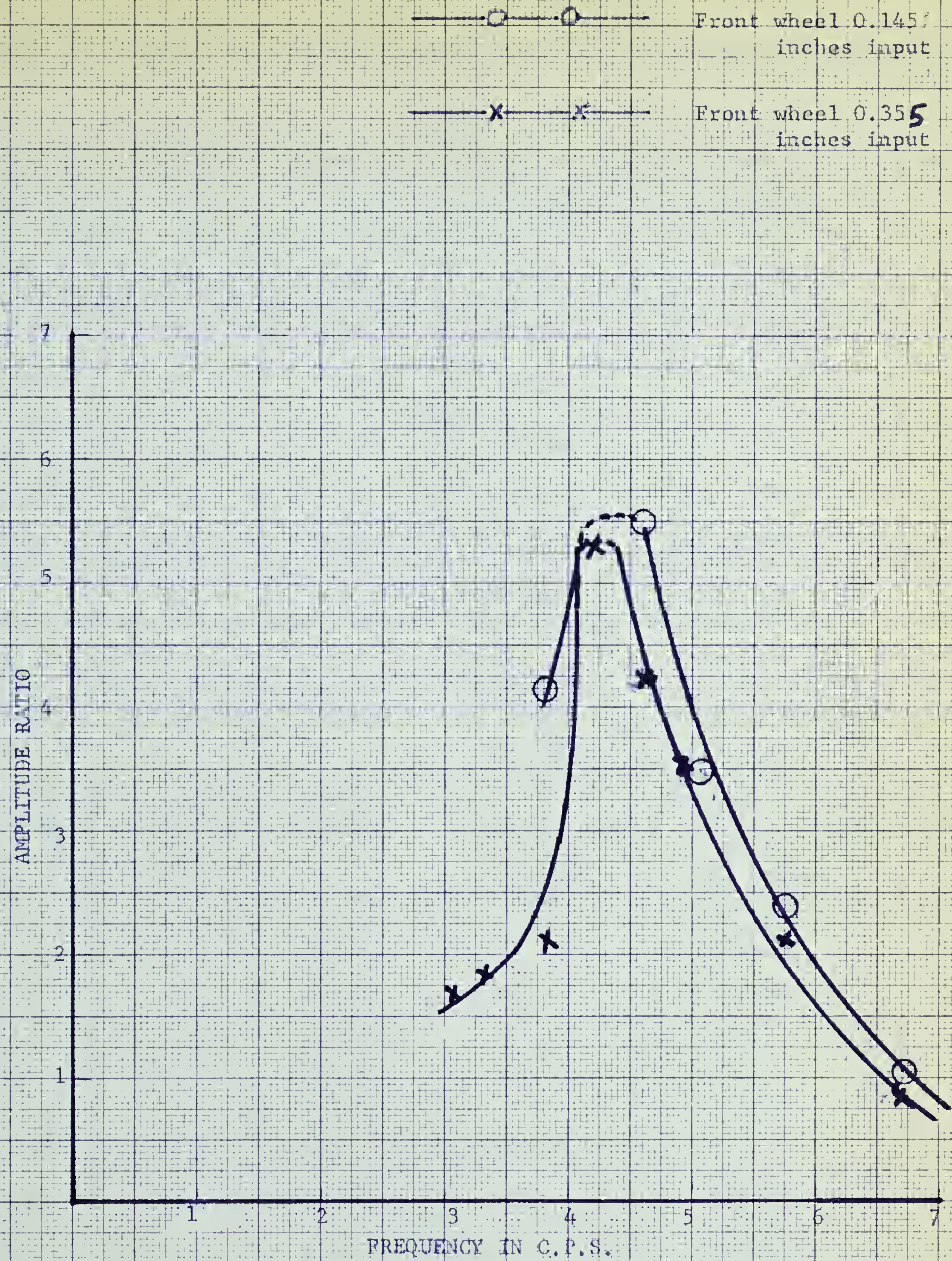
Results of analysis of the movies taken using the second input system (Figure 7) are tabulated below.

Table 5: Front wheels. Tire air pressure 26 psig. Input amplitude 0.145 inches

S. No.	Speed in rpm	Frequency in cps	Peak to Peak input amplitude in inches	Peak to Peak output amplitude in inches	Amplitude ratio
1	230	3.83	0.145	0.600	4.12
2	276	4.60	0.145	0.800	5.50
3	307	5.12	0.145	0.500	3.44
4	345	5.75	0.145	0.350	2.40
5	402	6.70	0.145	0.150	1.03

Table 6: Front wheels. Tire air pressure 26 psig. Input amplitude 0.355 inches

S. No.	Speed in rpm	Frequency in cps	Peak to Peak input amplitude in inches	Peak to Peak output amplitude in inches	Amplitude ratio
1	184	3.07	0.355	0.600	1.69
2	197	3.30	0.355	0.650	1.83
3	230	3.83	0.355	0.750	2.11
4	251	4.18	0.355	1.88	5.28
5	276	4.60	0.355	1.50	4.23
6	296	4.94	0.355	1.25	3.52
7	324	5.40	0.355	1.00	2.82
8	345	5.75	0.355	0.750	2.11
9	402	6.70	0.355	0.300	0.850



Graph 2: Frequency Versus Amplitude Ratio Curve for Front Wheels.

Table 7: Tractor on 100% air filled rear tires. Tire air pressure 12 psig

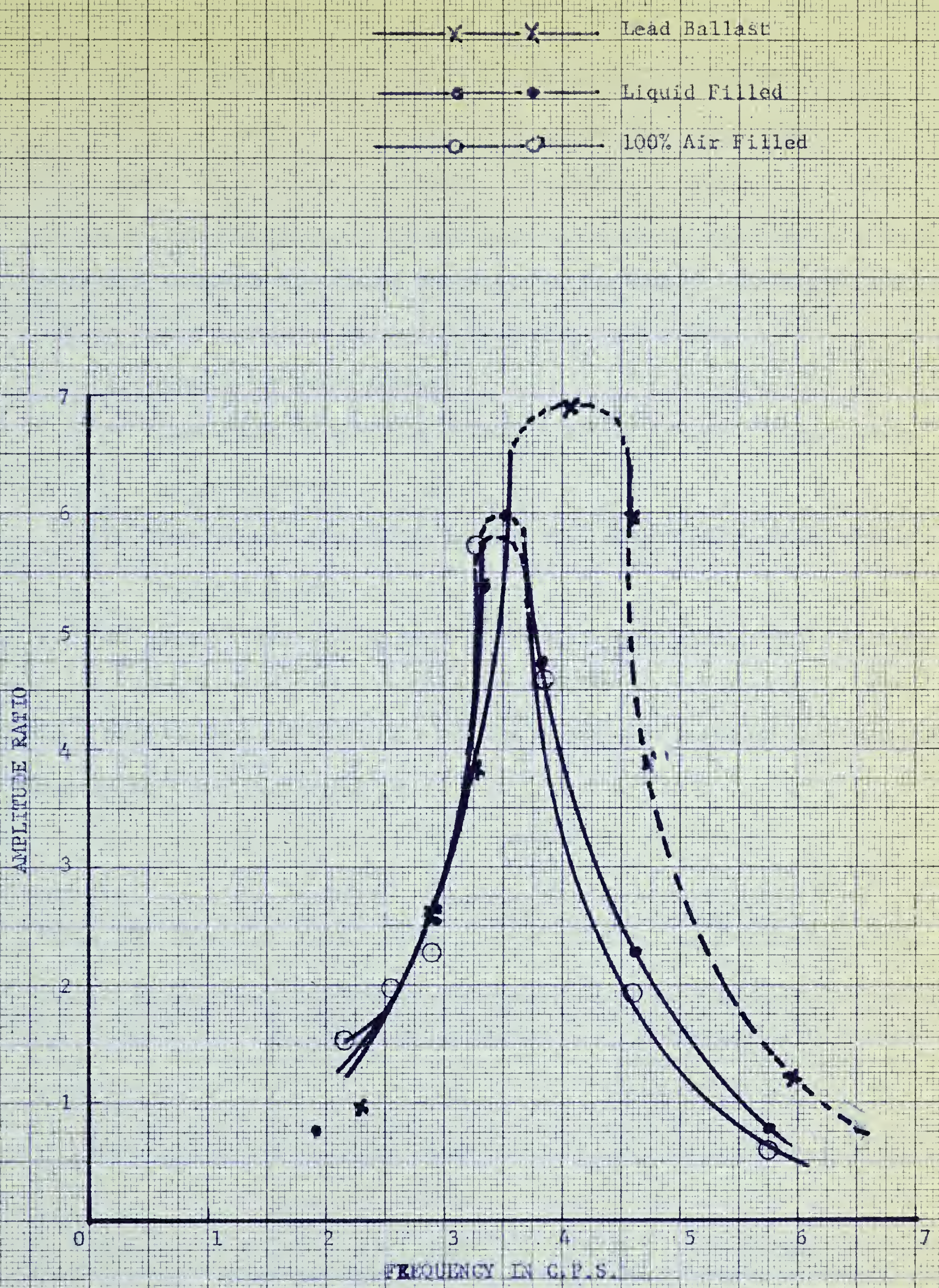
S. No.	Speed in rpm	Frequency in cps	Peak to Peak input amplitude in inches	Peak to Peak output amplitude in inches	Amplitude ratio
1	138	2.30	0.327	0.500	1.53
2	153	2.56	0.327	0.650	1.99
3	172	2.88	0.327	0.750	2.29
4	197	3.28	0.327	1.87	5.74
5	230	3.84	0.327	1.50	4.58
6	276	4.60	0.327	0.625	1.91
7	345	5.75	0.327	0.200	0.611

Table 8: Tractor on 75% liquid filled rear tires. Tire air pressure 12 psig

S. No.	Speed in rpm	Frequency in cps	Peak to Peak input amplitude in inches	Peak to Peak output amplitude in inches	Amplitude ratio
1	115	1.92	0.327	0.250	0.765
2	172	2.88	0.327	0.850	2.60
3	197	3.28	0.327	1.75	5.35
4	213	3.55	0.327	1.95	5.96
5	230	3.84	0.327	1.55	4.74
6	276	4.60	0.327	0.750	2.29
7	345	5.75	0.327	0.250	0.765

Table 9: Tractor on lead ballast tires.

S. No.	Speed in rpm	Frequency in cps	Peak to Peak input amplitude in inches	Peak to Peak output amplitude in inches	Amplitude ratio
1	138	2.30	0.210	0.200	0.953
2	172	2.88	0.210	0.550	2.62
3	197	3.28	0.210	0.800	3.81
4	244	4.07	0.210	1.45	6.90
5	276	4.60	0.210	1.25	5.95
6	356	5.94	0.210	0.250	1.19



Graph 3: Amplitude Ratio Versus Frequency Curve for Three Types of Rear Tires Considered.

4.5. DETERMINATION OF STATIC SPRING RATE OF TIRE WITH 100% AIR

The experiment was conducted to determine the static spring rate of the rear tires (100% air filled) at 23 psig and 12 psig. It was felt that the static spring rate on level ground would be different than when a tractor actually hits bumps and rides on the bumps. The static spring rate test was conducted under the following conditions:

1. Tractor on level ground.
2. Rear wheels resting on 6" wide x 6" long block.* Front wheels on level ground.
3. Rear wheels resting on 6" wide x 4" long block. Front wheels on level ground.
4. Rear wheels resting on 6" wide x 2" long block. Front wheels on level ground.

Under each condition the tire was loaded incrementally and each time the deflections were measured.

The data observed are presented in tabular form below.

Table 10: The Rear Wheels on Level Ground Tire Air Pressure 23 psig

S. No.	Weight in Pounds	Deflection in Inches
1	520	3/16
2	1100	5/16
3	1800	1/2
4	2500	11/16

* All blocks were 3 5/8 inches high.

Table 11: The Rear Wheels Resting on 6"x6" Blocks - Tire Air Pressure 23 psig

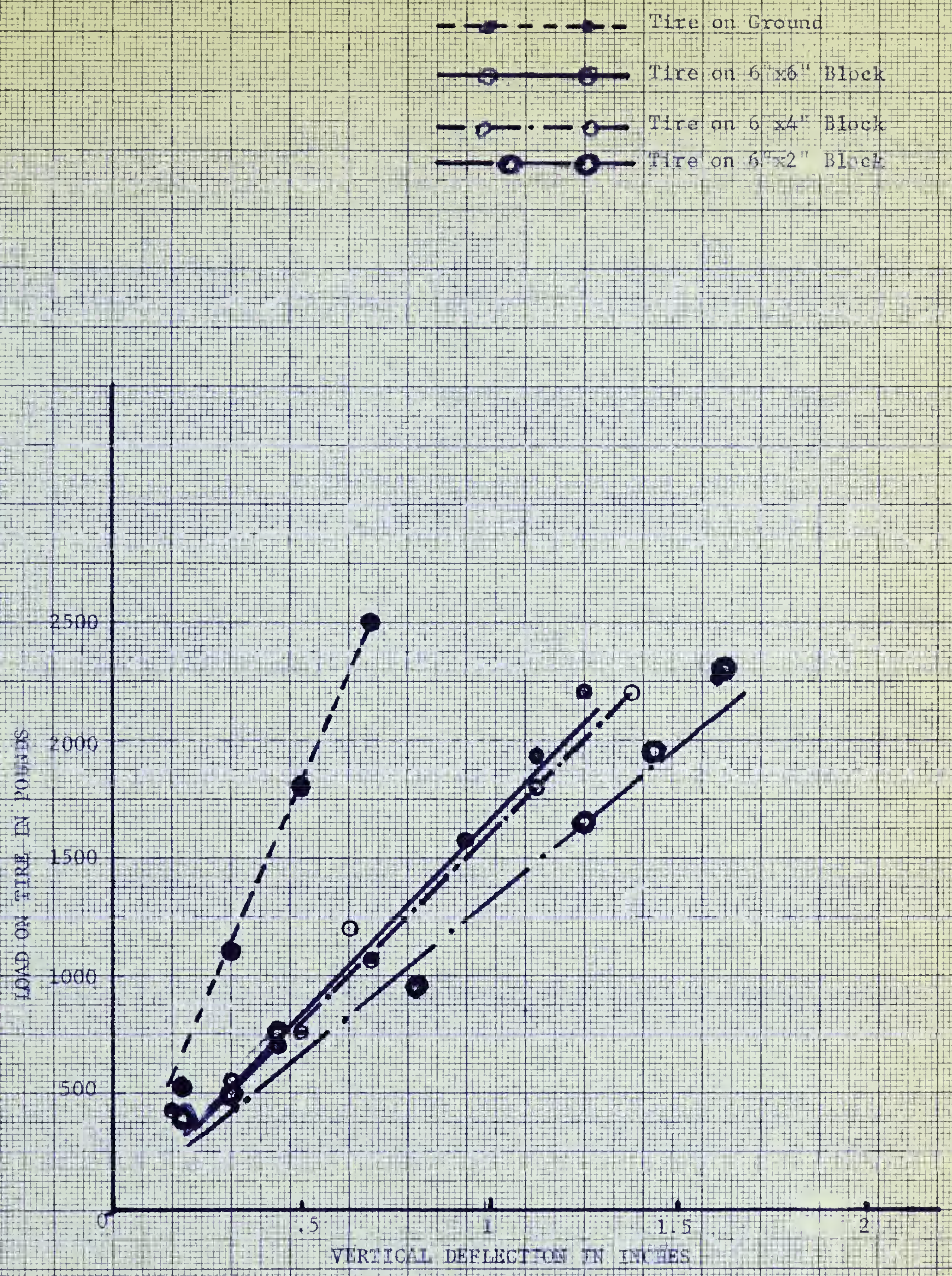
S. No.	Weight in Pounds	Deflection in Inches
1	425	5/16
2	750	1/2
3	1075	11/16
4	1565	15/16
5	1930	1 1/8
6	2200	1 1/4

Table 12: The Rear Wheels Resting on 6"x4" Blocks - Tire Air Pressure 23 psig

S. No.	Weight in Pounds	Deflection in Inches
1	400	3/16
2	550	5/16
3	700	7/16
4	1200	5/8
5	1800	1 1/8
6	2200	1 3/8

Table 13: The Rear Wheels Resting on 6"x2" Blocks - Tire Air Pressure 23 psig

S. No.	Weight in Pounds	Deflection in Inches
1	500	5/16
2	750	7/16
3	950	13/16
4	1650	1 1/4
5	1950	1 7/16
6	2300	1 5/8



Graph 4: Comparison of Load Versus Deflection Curves for 100% Air Filled Tire at 23 psig Pressure.

Table 14: The Rear Wheels Resting on Level Ground - Tire Air Pressure 12 psig

S. No.	Weight in Pounds	Deflection in Inches
1	400	3/16
2	750	5/16
3	1100	7/16
4	1470	19/32
5	1950	13/16
6	2310	15/16

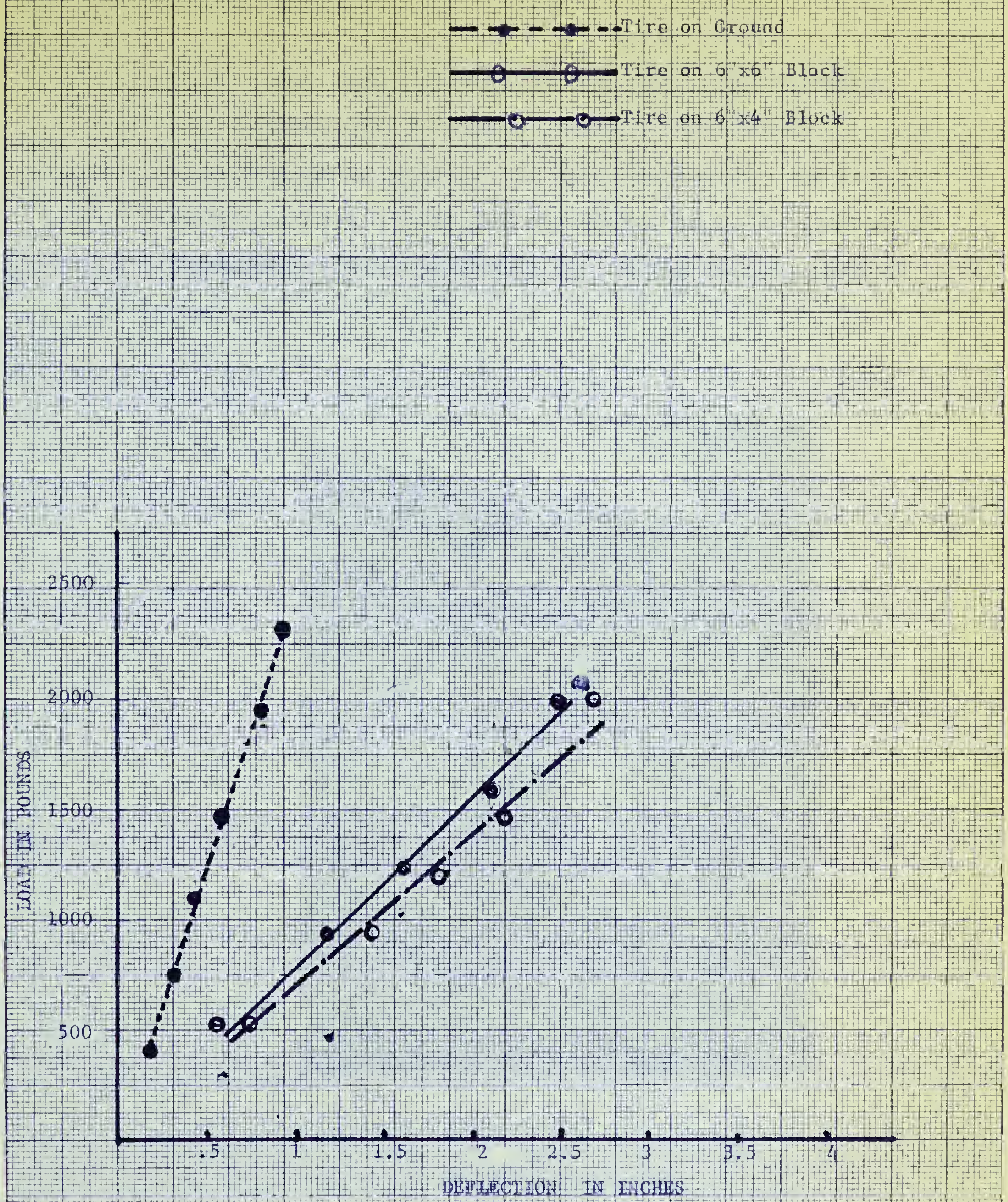
Table 15: The Rear Wheels Resting on 6"x6" Blocks - Tire Air Pressure 12 psig

S. No.	Weight in Pounds	Deflection in Inches
1	525	9/16
2	940	1 3/16
3	1240	1 5/8
4	1590	2 1/8
5	2000	2 1/2

Table 16: The Rear Wheels Resting on 6"x4" Blocks - Tire Air Pressure 12 psig

S. No.	Weight in Pounds	Deflection in Inches
1	520	3/4
2	940	1 7/16
3	1200	1 13/16
4	1470	2 3/16
5	2000	2 11/16

Due to certain technical difficulties it was not possible to take observations when the tractor rear wheels were resting on blocks of 6" x 2" size and the tire pressure was 12 psig.



Graph 5: Comparison of Load Versus Deflection Curves for 100% Air Filled Tire at a Pressure of 12 psig.

4.6. MASS AND MASS MOMENT OF INERTIA OF THE TRACTOR

For simulating the system equations it was necessary to know the mass and mass moment of inertia of the tractor. The tractor weight was taken from the Nebraska Tractor Test. The tractor was suspended as a compound pendulum for finding the mass moment of inertia.

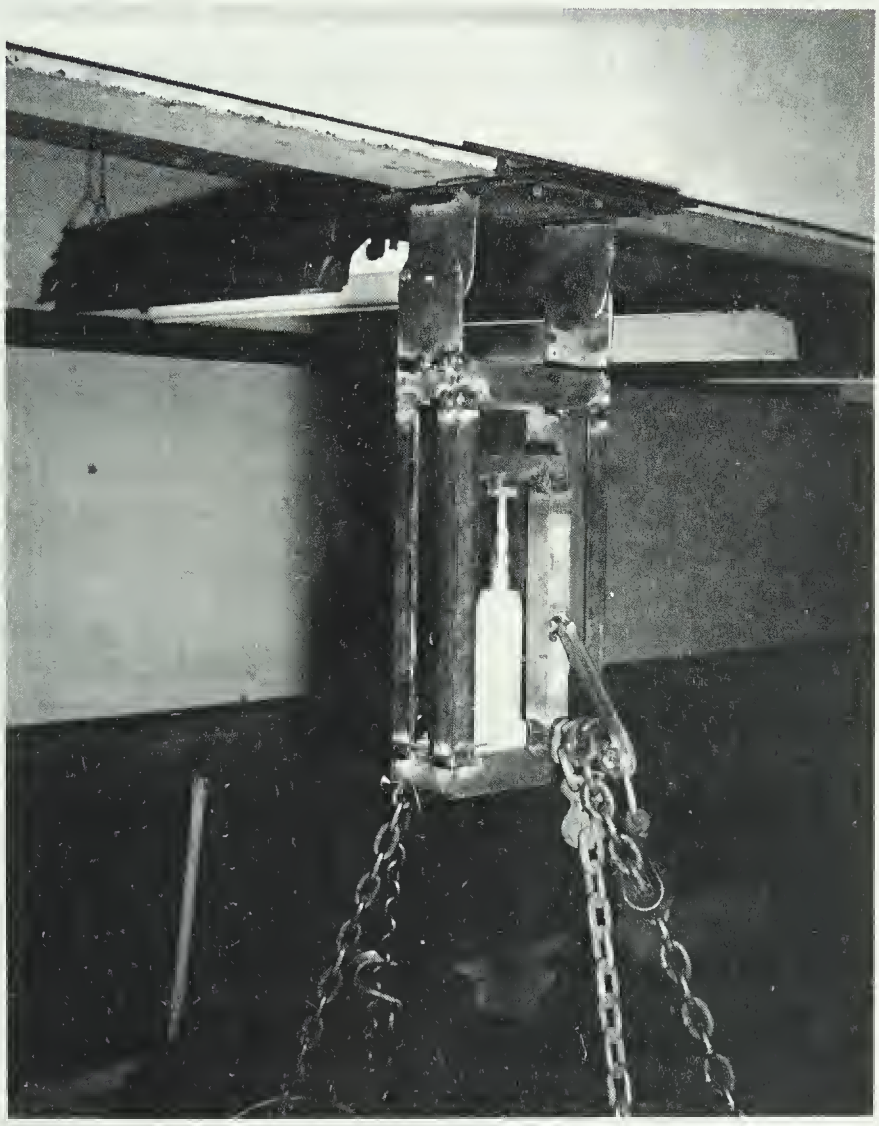


Figure 8: Tractor Suspended in Air.

The mass moment of inertia was calculated by using the formula:

$$I.C.G. = W \left[r \left(\frac{T}{2\pi}\right)^2 - \frac{r^2}{g} \right]$$

Where

I.C.G. = mass moment of inertia.

W = weight of tractor.

r = radius in feet from the centre of gravity to the pivot point.

T = time taken in seconds to complete one cycle.

g = acceleration due to gravity.

The following observations were made when the tractor was swung at the pivot point.

a. Tractor swinging with respect to its longitudinal axis.

W = 3390 lbs.

r = 8.29 ft.

T = 3.30 sec/cycle

$$I.C.G.\theta = 3390 \left[8.29 \times \left(\frac{3.30}{2\pi}\right)^2 - \frac{8.29^2}{32.2} \right]$$

$$= 523 \text{ lb-ft-sec}^2$$

b. Tractor swinging with respect to its transverse axis.

W = 3390 lbs.

r = 8.38 ft.

T = 3.24 sec.

$$I.C.G.\Phi = 3390 \left[8.38 \left(\frac{3.24}{2\pi}\right)^2 - \frac{8.38^2}{32.2} \right]$$

$$= 168 \text{ lb-ft-sec}^2$$

The following values of mass and mass moment of inertia were used for solving the system equations.

Table 17: Mass and mass moment of inertia of tractor.

S. No.	Type of Tire Ballast	Mass $lb\text{-}sec^2/ft$	$I\Theta$ $lb\text{-}ft\text{-}sec^2$	$I\Phi$ $lb\text{-}ft\text{-}sec^2$
1	100% Air Filled	110.8	523	168
2	75% Liquid	136.8	622	308
3	Lead Ballast	154.2	651	402

$I\Theta$ and $I\Phi$ have an accuracy of $\pm$ 10%.

5. AN ANALOG COMPUTER SIMULATION AND REPRODUCTION OF FREQUENCY VERSUS AMPLITUDE RATIO CURVES

An analog computer was used to solve the system equations. A Massey Ferguson Gasoline tractor was used to find the tractor coefficients.

5.1 CIRCUIT DIAGRAMS FOR COMPUTER

The system equations were set up on an analog computer. The following block diagram represented the system equation No. 3 on the computer.

Equation No. 3

$$\ddot{X}_1 = - k_5\dot{X}_1 - K_6(\dot{X}_2 + \dot{X}_3) - K_7(X_1) - K_8(X_2 + X_3) - K_9(\ddot{X}_2 + \ddot{X}_3)$$

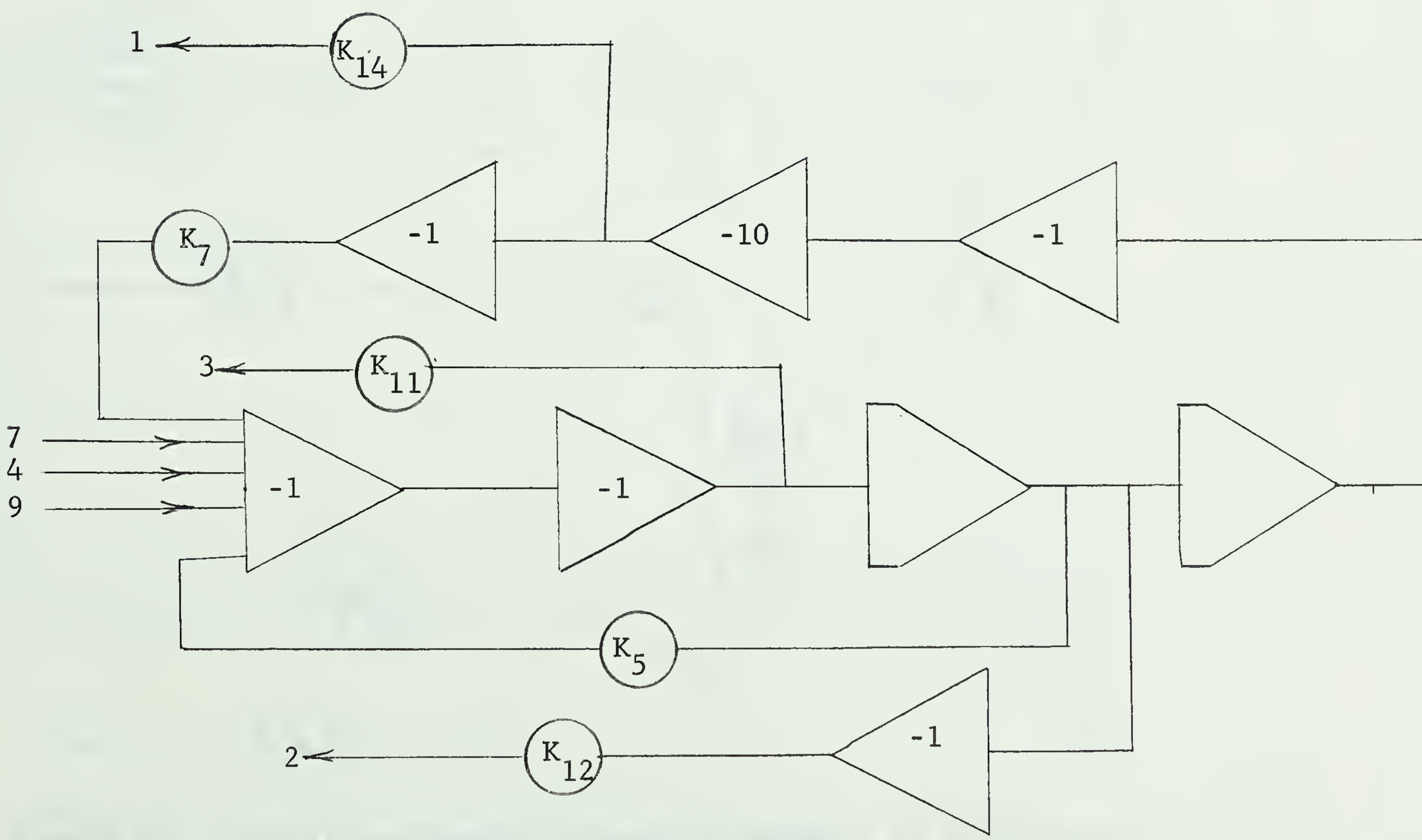


Figure 9: Circuit Representing Equation Number 3 on the Computer.

The system equation Number 4 was set up on the computer as shown in the following circuit diagram.

Equation No. 4

$$\ddot{X}_2 = -K_{10}\ddot{X}_3 + K_{11}\ddot{X}_1 + K_{12}\dot{X}_1 - K_{13}(\dot{X}_2 + \dot{X}_3) + K_{14}(X_1) - K_{15}\left\{(X_2) + (X_3)\right\}$$

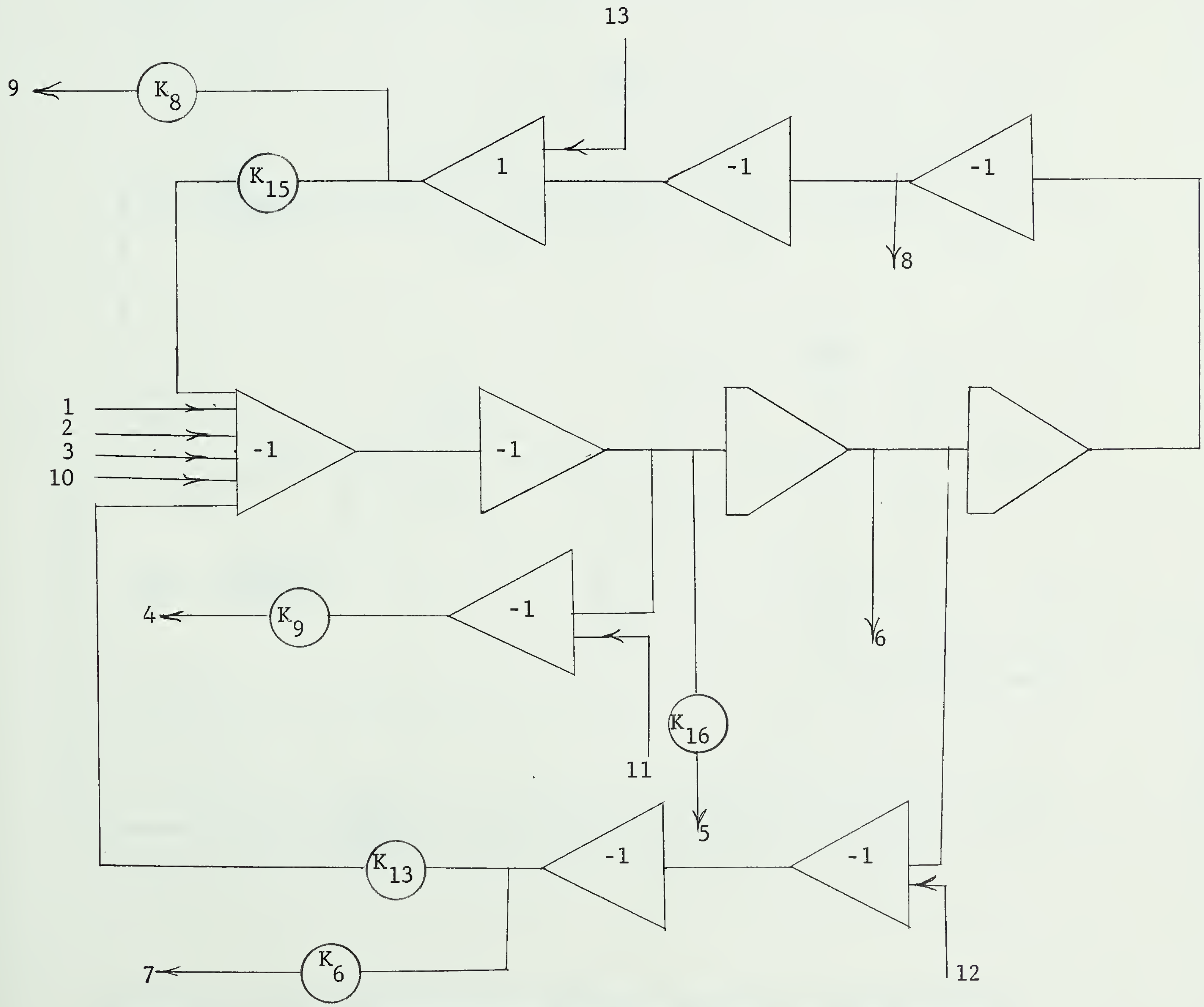


Figure 10: Circuit Representing Equation Number 4 on the Computer.

System Equation Number 5 was set up on the computer as shown in the following circuit diagram.

Equation No. 5

$$\ddot{X}_3 = K_{16}\ddot{X}_2 - K_{17}(\dot{X}_3 - \dot{X}_2) - K_{18}\left\{ (X_3) - (X_2) \right\}$$

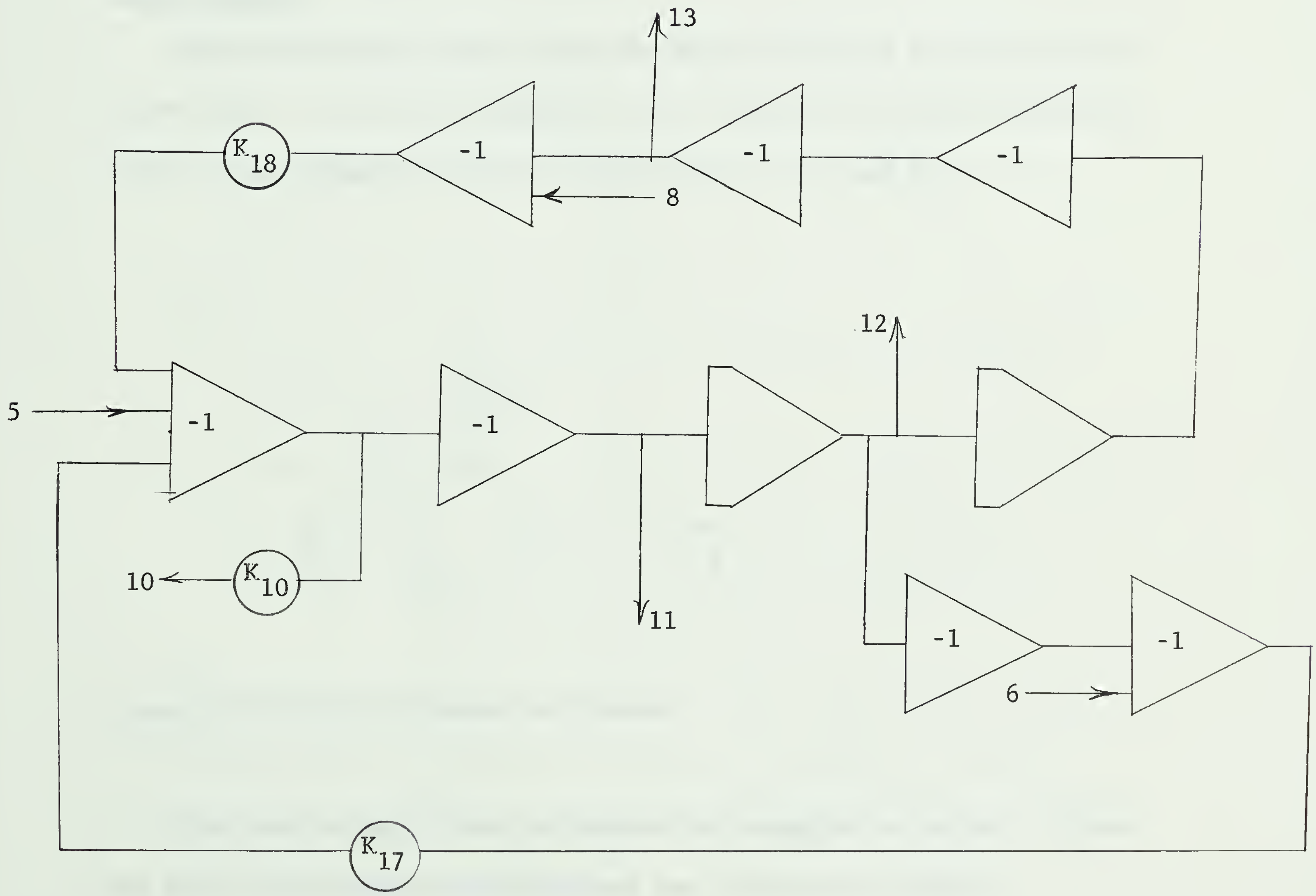


Figure 11: Circuit Representing Equation Number 5 on the Computer.

Potentiometer coefficients were calculated by using predetermined values for the Massey Ferguson 135 gasoline tractor. As the scale factor was 500, all coefficients were divided by 500 except those involving dampings. All damping coefficients were divided by $\sqrt{500}$, or, 22.36.

5.1.1 Relay

Relays were used on all integrators to permit control over the integrating time of the integrators.

5.1.2 Diodes

When the tractor tire leaves the ground there is no spring force in the tires. This was simulated on the computer by putting diodes on those three integrators whose output were X_1, X_2, and X_3.

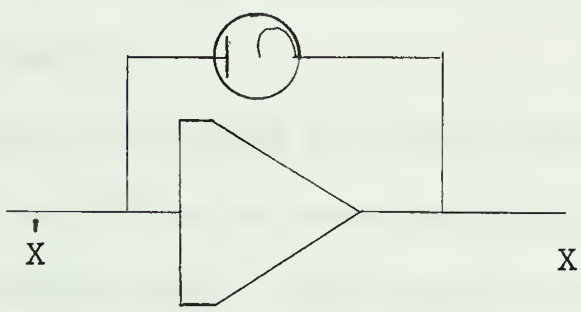


Figure 12: Circuit Diagram for Diodes

The above circuit does not permit any negative value of X. This was done because upward displacement was considered negative.

5.2 INPUT TO COMPUTER

For testing the effect of the tire ballasts on the dynamic behaviour of the tractor, it was essential to give the same input to the computer each time. For this reason a new random input to the computer for each run was not feasible. Instead, random bumps were simulated on magnetic tapes, and the same tapes were used as inputs for each run. By using random tables two tracks about a quarter of a mile long were simulated. The following assumptions were made in simulating the track for the tractor.

1. The tractor might hit bumps of the following heights.

   1",2",3",4",5",6".

2. The tractor might pass over depressions of the following depths.

   1",2",3",4",5",6".

3. There was an equal opportunity of passing over a bump or depression.
4. Shape of the bumps and depressions were assumed to be square.
5. Any one of the following spacings might occur between successive bumps, depressions or bumps and depressions.

   2',4',6',8',10',12',14',16'.

Two columns of the random table were used, one for deciding the size of the bump or hollow and the other for deciding the spacing between them. It was also assumed that even numbers represent bumps and odd numbers represent hollows.

Bumps and hollows were recorded on the tape in 400 cycle, amplitude modulated signals. Rectifiers were used to change the AC output of the tapes into DC.

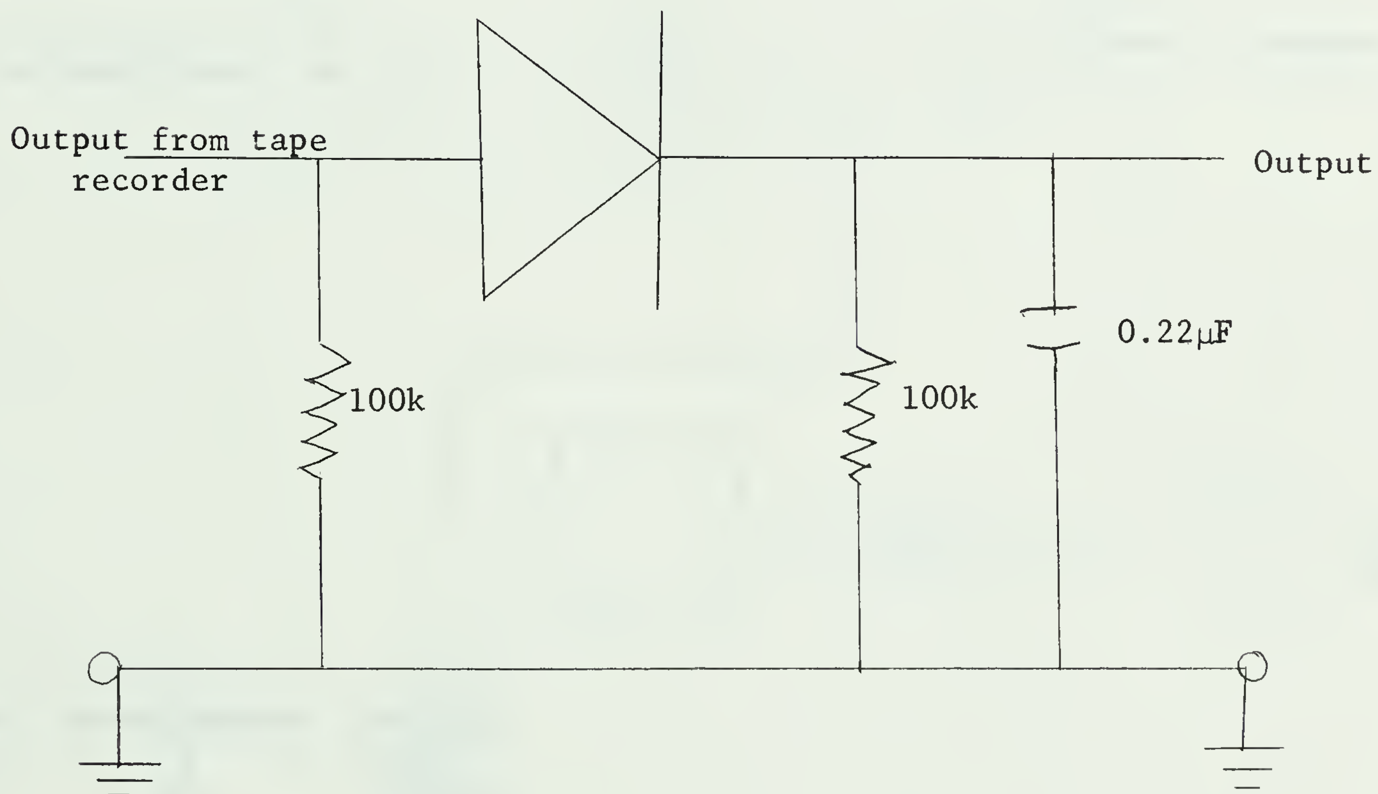
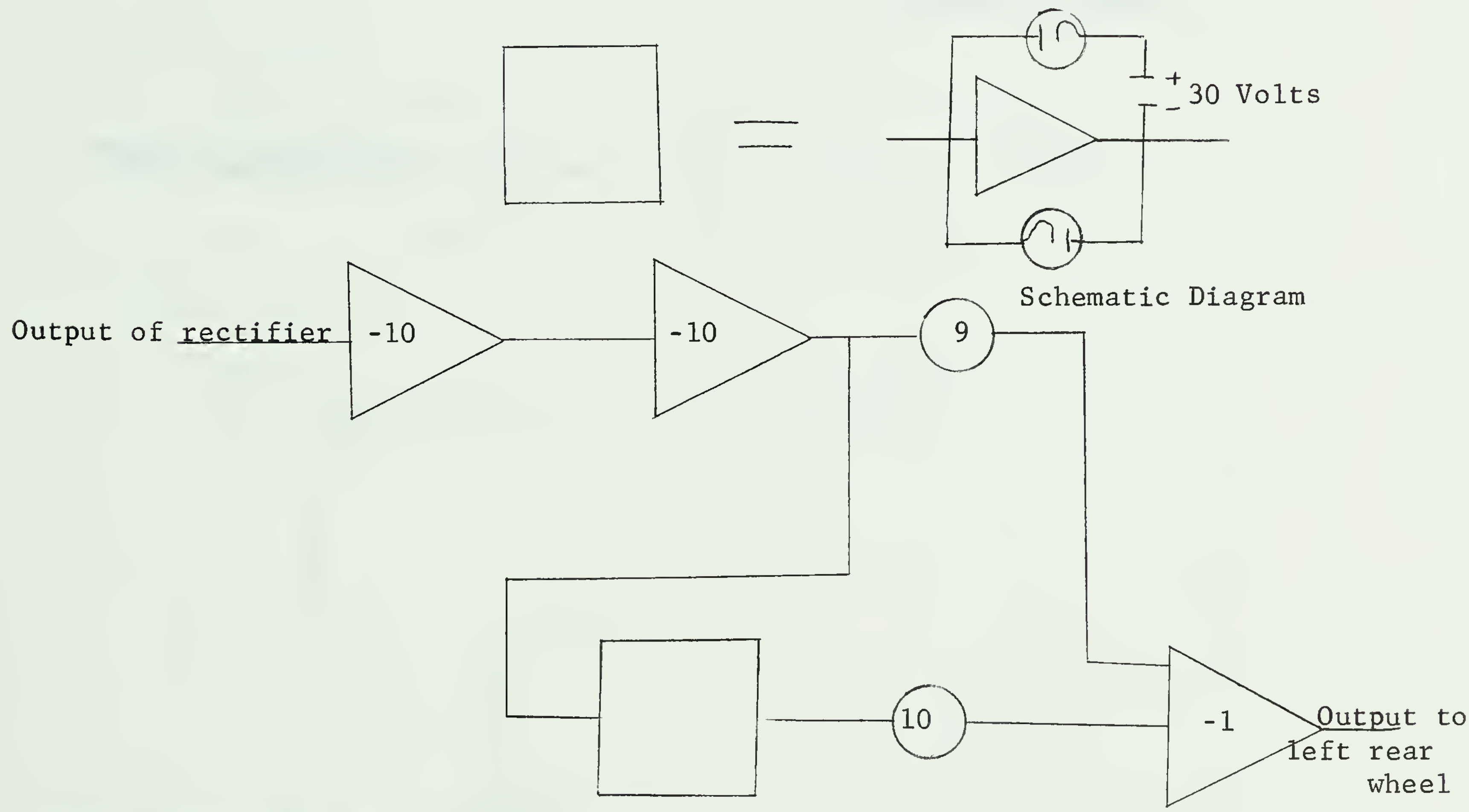


Figure 13: Circuit Diagram of Rectifier.

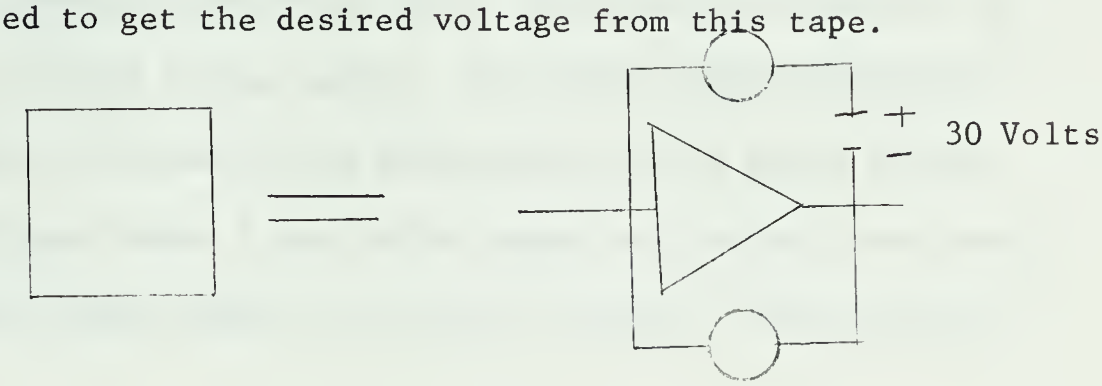
Bumps were represented by positive voltage and depressions were represented by negative voltages. Each inch of bump or depression was represented by plus or minus 0.25 volts respectively. Because the tape output was always positive and only .75V was the maximum output of Tape Number 2 the following computer circuit was used to obtain both the bumps and hollows.



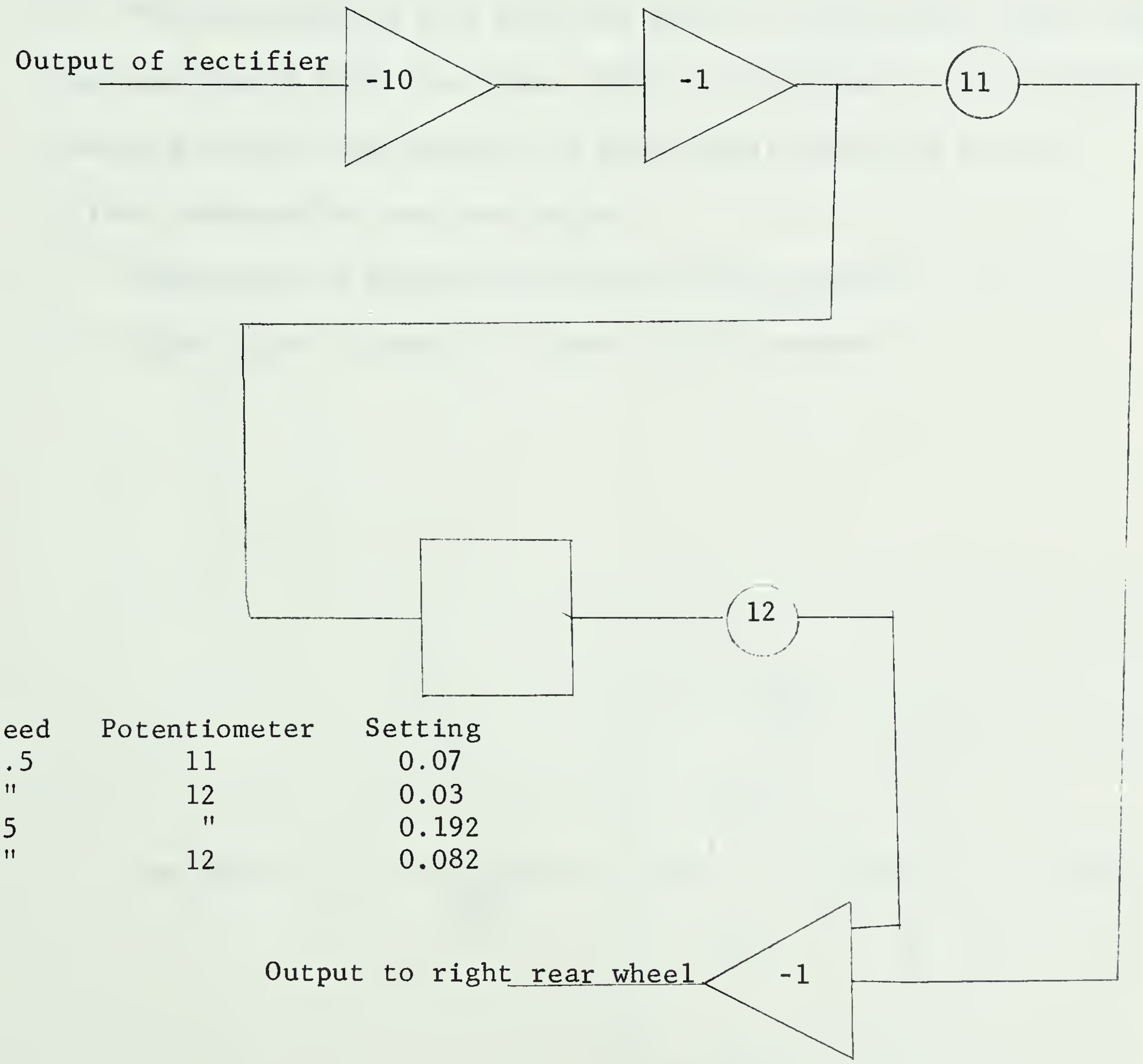
Speed	Potentiometer	Setting
7.5	9	0.07
"	10	0.024
15	9	0.192
"	10	0.066

Figure 14: Circuit Diagram for Obtaining Bumps and Depressions from Tape Number 2.

The maximum output from Tape Number 1 was 6 volts. The following circuit was used to get the desired voltage from this tape.



Schematic Diagram



Speed	Potentiometer	Setting
7.5	11	0.07
"	12	0.03
15	"	0.192
"	12	0.082

Figure 15: Circuit Diagram for Obtaining Bumps and Depressions from Tape Number 1.

Outputs of the tapes were summed and divided by two to get the input for the centre of the front axle. This was done because the front axle is pivoted at the centre. The pivot point's deflection is always half of the sum of the deflections of the front wheels. The output of Tape Number 2 was fed as input to the left rear wheels and the output of Tape Number 1 was fed as input to the right rear wheels.

5.2.1 Time Lag

The front wheels pass over the bumps or depressions first, then the rear wheels pass over them. This was obtained by calculating the time lag at both the speeds (7.5 and 15 mph) and using low pass filter circuits for the time delay.

Time lag at a speed of 7.5 mph = 0.545 seconds.

Time lag at a speed of 15 mph = 0.272 seconds.

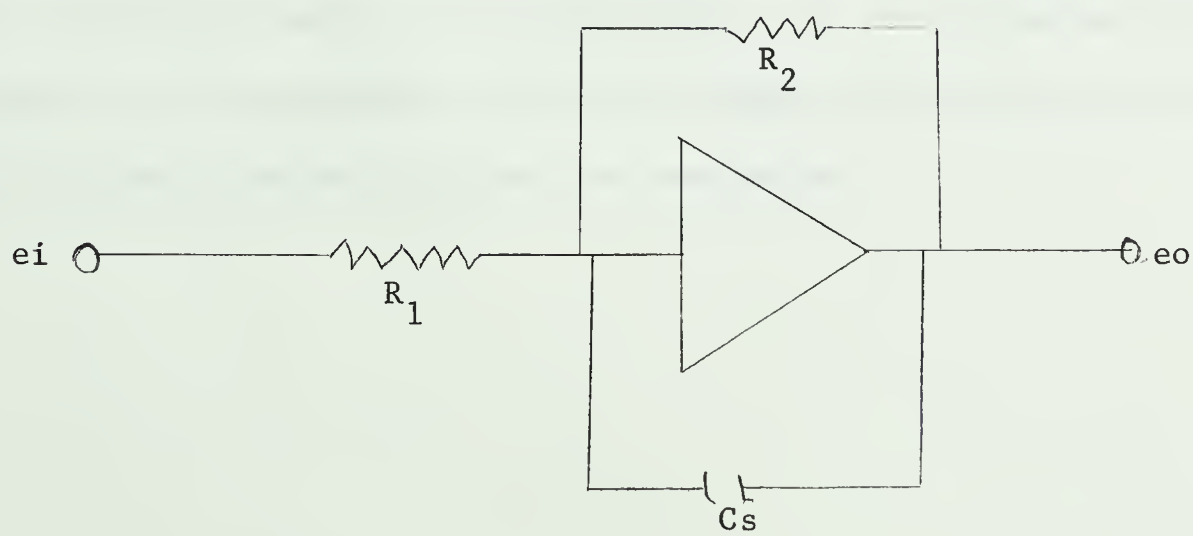


Figure 16: Circuit Diagram of a Low Pass Filter.

$$eo = -\frac{R_2}{R_1}\,\frac{1}{R_2C_s + 1}\,ei$$

Where time lag in seconds is:-

$$\frac{R_2}{R_1}\,\frac{1}{R_2C_s + 1}$$

The following size of resistors and capacitors were used to obtain 0.545 seconds time lag.

R_1 = 610,000 ohms

R_2 = 500,000 ohms

C = 1 microferad

The following size resistors and capacitors were used to obtain 0.272 seconds time lag.

R_1 = 330 k ohms

R_2 = 100 k ohms

C = 1 microferad.

The outputs of the previous circuits passed through the time delay and then were fed as inputs to the rear wheels. It is recognized that the low-pass filter appreciably altered the shape of the bumps and hollows. However, no other time delay circuit was available.

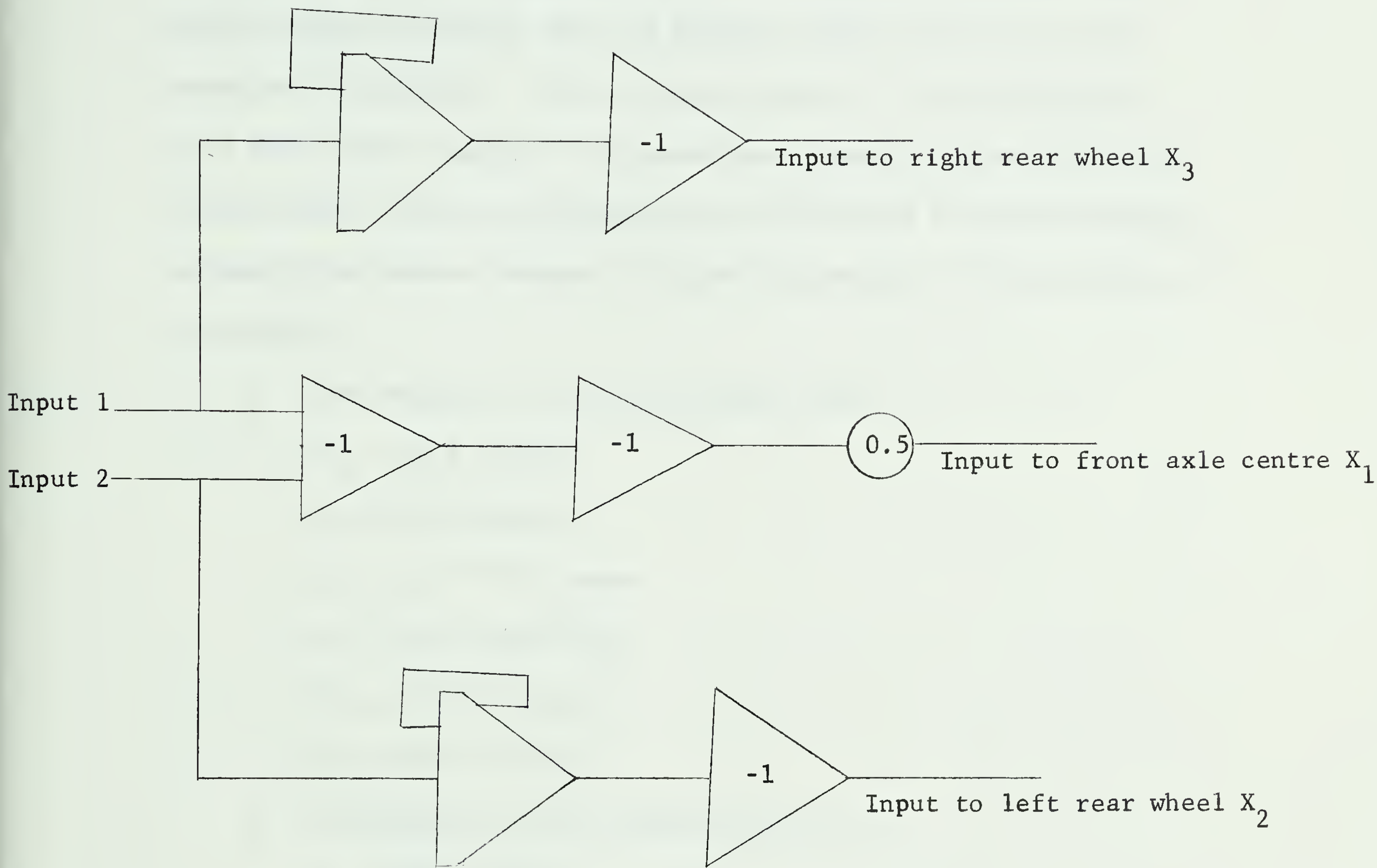


Figure 17: Circuit Diagram for Time Lag.

5.3 DETERMINING THE DYNAMIC SPRING AND DAMPING RATE OF THE TIRES WITH AND WITHOUT BALLAST.

The system equations were simulated on an electronic analog computer as shown in Figures 9,10,11. All system parameters were known except the spring rate and damping before the curve on the computer was produced. The following values of the coefficients were used on the computer for generating frequency versus amplitude ratio curves. These coefficients were calculated by using parameter values of the Massey-Ferguson 135 gasoline tractor. These values are as follows:

1. The tractor on 100% air filled tire*.

   L_1 = 42.8 inches

   L_2 = 29.2 inches

   $L_3 = L_4$ = 28.2 inches

   m = 110.8 $\text{lb.sec}^2/\text{ft}$

   $I\Theta$ = 523 lb.ft.sec^2

   $I\Phi$ = 168 lb.ft.sec^2

2. The tractor on 75% liquid filled tire.

   L_1 = 48.6 inches

   L_2 = 23.4 inches

   $L_3 = L_4$ = 28.2 inches

   m = 136.8 $\text{lb.sec}^2/\text{ft}$

   $I\Theta$ = 622 lb.ft.sec^2

   $I\Phi$ = 308 lb.ft.sec^2

* All rear tires pressure 12 psig .
Front tire pressure 26 psig.

3. The tractor on lead ballast tire.

$L_1 = 51.3$ inches

$L_2 = 20.6$ inches

$L_3 = L_4 = 28.2$ inches

$m = 154.2$ lb.sec^2/ft.

$I\Theta = 651$ lb.ft.sec^2

$I\Phi = 402$ lb.ft.sec^2

First the frequency versus amplitude ratio curve for the front wheels was simulated, a sinusoidal input was then applied, and remaining parameters obtained by adjusting potentiometer settings until simulated ratios were similar to the experimentally obtained ratios. The spring rate and damping rate were found to be as shown below.

$k_1 = 38400$ lb/ft

$c_1 = 121$ lb.sec/ft

The values of coefficients used in simulating the frequency versus amplitude ratio curve for the rear wheels on the computer are given below.

1. Tractor on 100% air filled tire.

$$k_5 = \frac{C_1}{mk_1} = \frac{121}{110.8 \times 0.406 \times (\text{scale factor})\ 1/2} = 0.121$$

A scale factor of 500 was used.

$k_7 = 1.73$

$k_9 = 0.001$

$k_{10} = 0.002$

$k_{11} = 0.004$

$k_{12} = 0.444$

$k_{14} = 6.29$

$k_{16} = 0.002$

Coefficient potentiometer readings representing the spring force and damping of the rear wheels were changed until the frequency versus amplitude ratio curve simulated on the computer was similar to the curve obtained experimentally (Graph No. 8). The following potentiometer settings were obtained.

$k_6 = 0.250$

$k_8 = 0.240$

$k_{13} = 0.626$

$k_{15} = 0.599$

$k_{17} = 0.716$

$k_{18} = 0.687$

2. Tractor on 75% liquid filled tires.

$$k_5 = \frac{C_1}{mk_1 \text{ (Scale factor)}} \; 1/_2 = \frac{121}{136.8 \times .325 \; (500)} \; 1/_2 = 0.122$$

Similarly

$k_7 = 1.73$

$k_9 = 0.002$

$k_{10} = 0.002$

$k_{11} = 0.004$

$k_{12} = 0.428$

$k_{14} = 6.07$

$k_{16} = 0.002$

The following potentiometer settings representing the spring rate and damping of rear wheels were obtained by simulation.

$k_6 = 0.333$

$k_8 = 0.299$

$k_{13} = 0.547$

$k_{15} = 0.490$

$k_{17} = 0.516$

$k_{18} = 0.462$

3. Tractor on lead ballast tire.

$$k_5 = \frac{C1}{mk_1(\text{scale factor})}^{1/2} = \frac{121}{154.2 \times 0.283\ (500)}^{1/2} = 0.124$$

$k_7 = 1.76$

$k_9 = 0.002$

$k_{10} = 0.002$

$k_{11} = 0.004$

$k_{12} = 0.430$

$k_{14} = 6.09$

$k_{16} = 0.002$

The following potentiometer settings representing the spring rate and damping of rear wheels were finally obtained by simulation.

$k_6 = 0.477$

$k_8 = 0.526$

$k_{13} = 0.650$

$k_{15} = 0.700$

$k_{17} = 0.556$

$k_{18} = 0.599$

These potentiometer settings were used while running the actual programme except as indicated in "Results and Discussion" on Page 79.

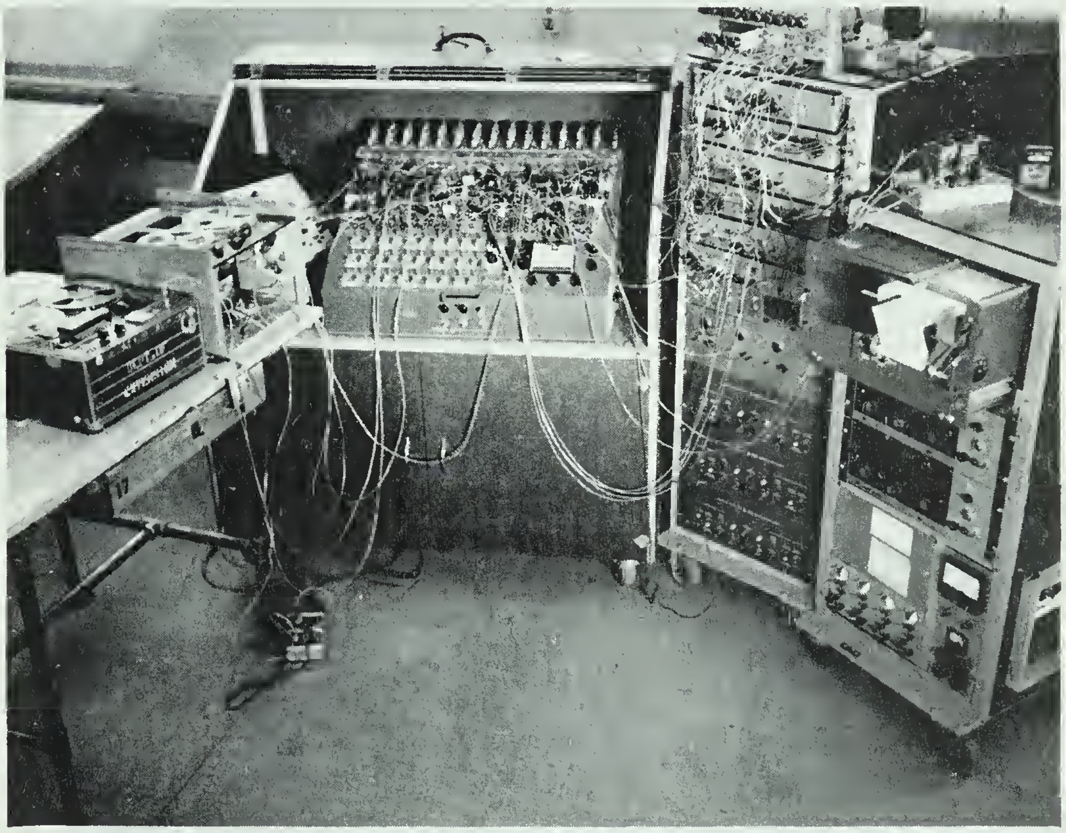


Figure 18: Programme Circuit on the Computer.

Comparison of the curves obtained experimentally and those produced on the computer are shown in the graphs that follow.

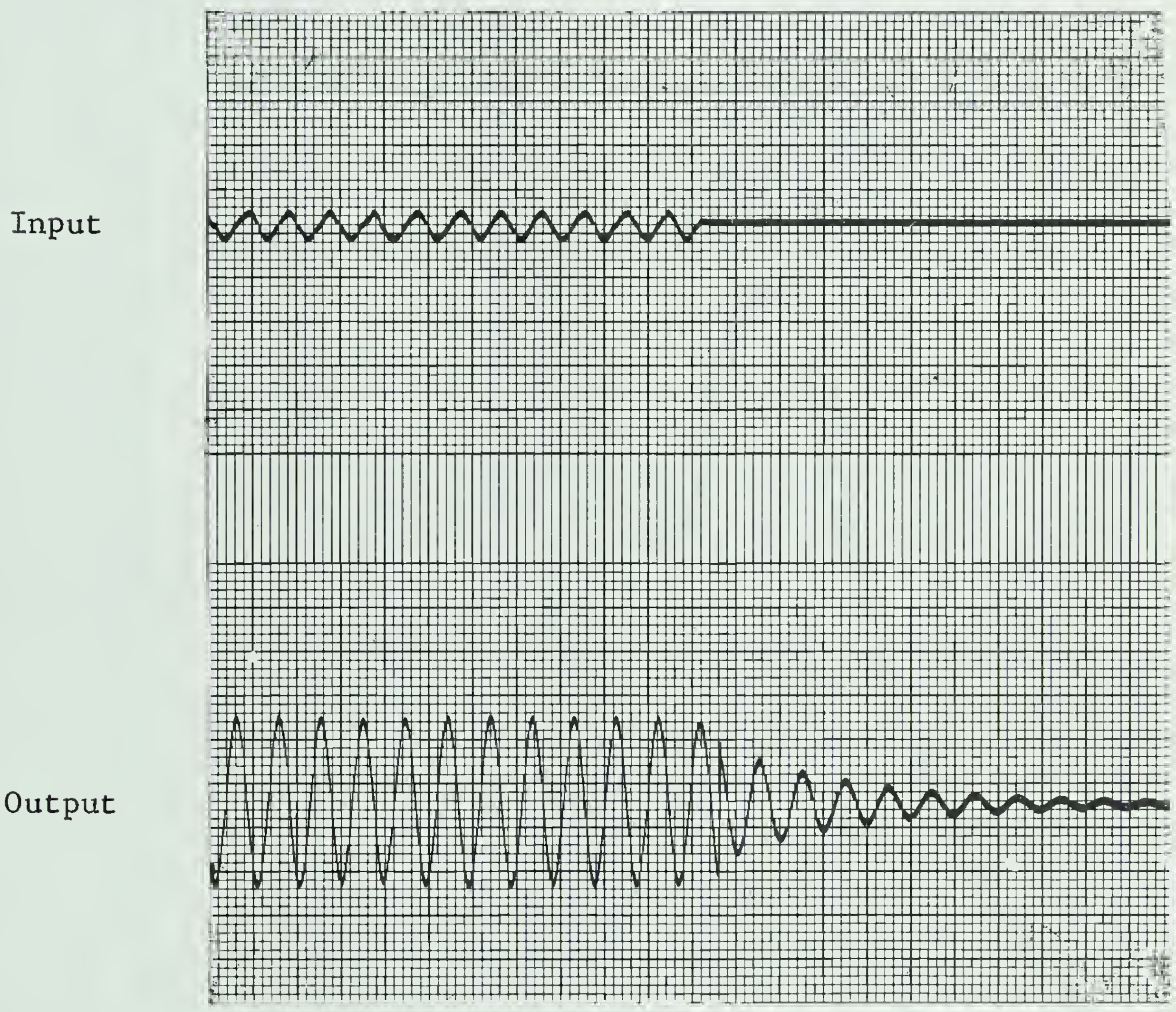
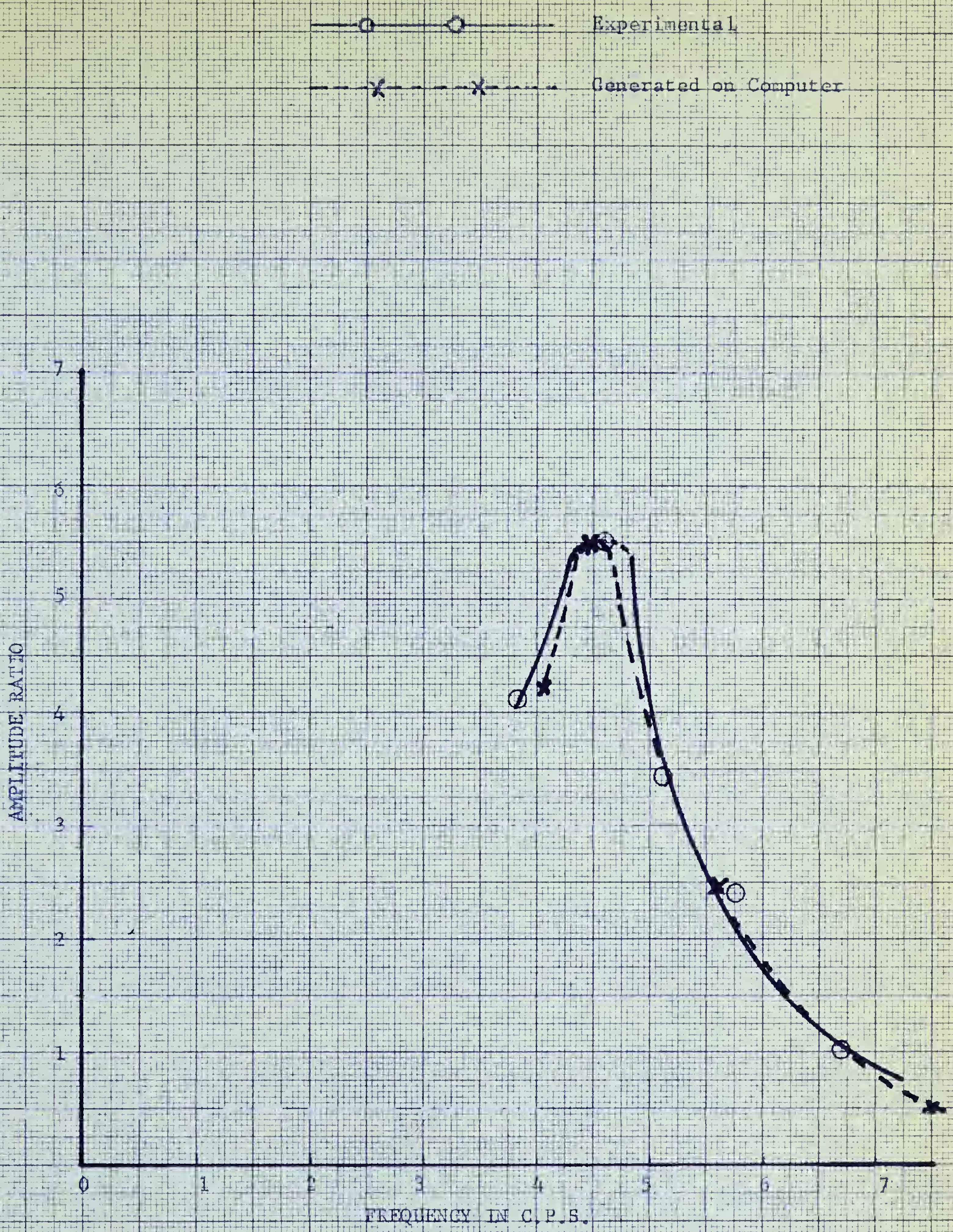
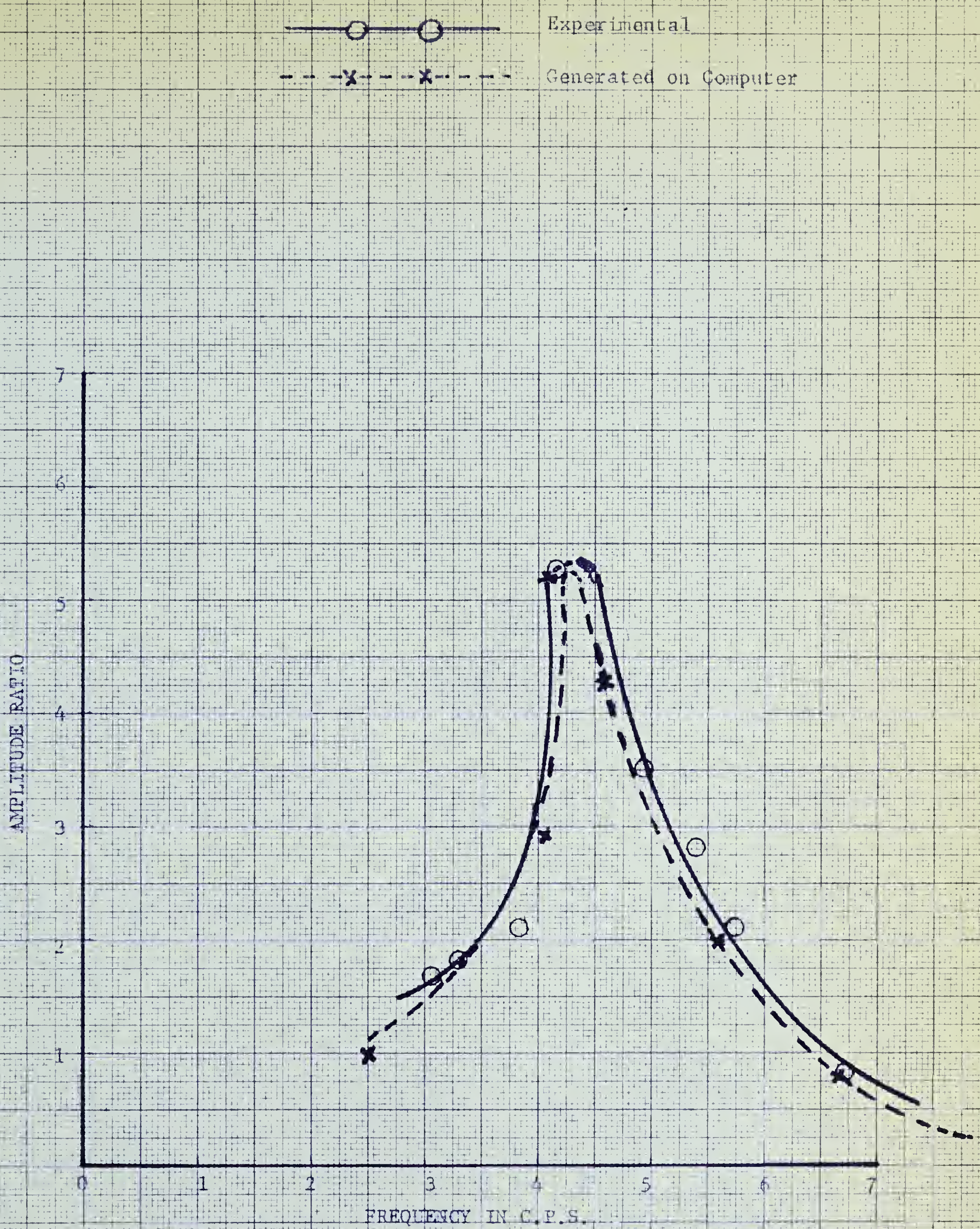


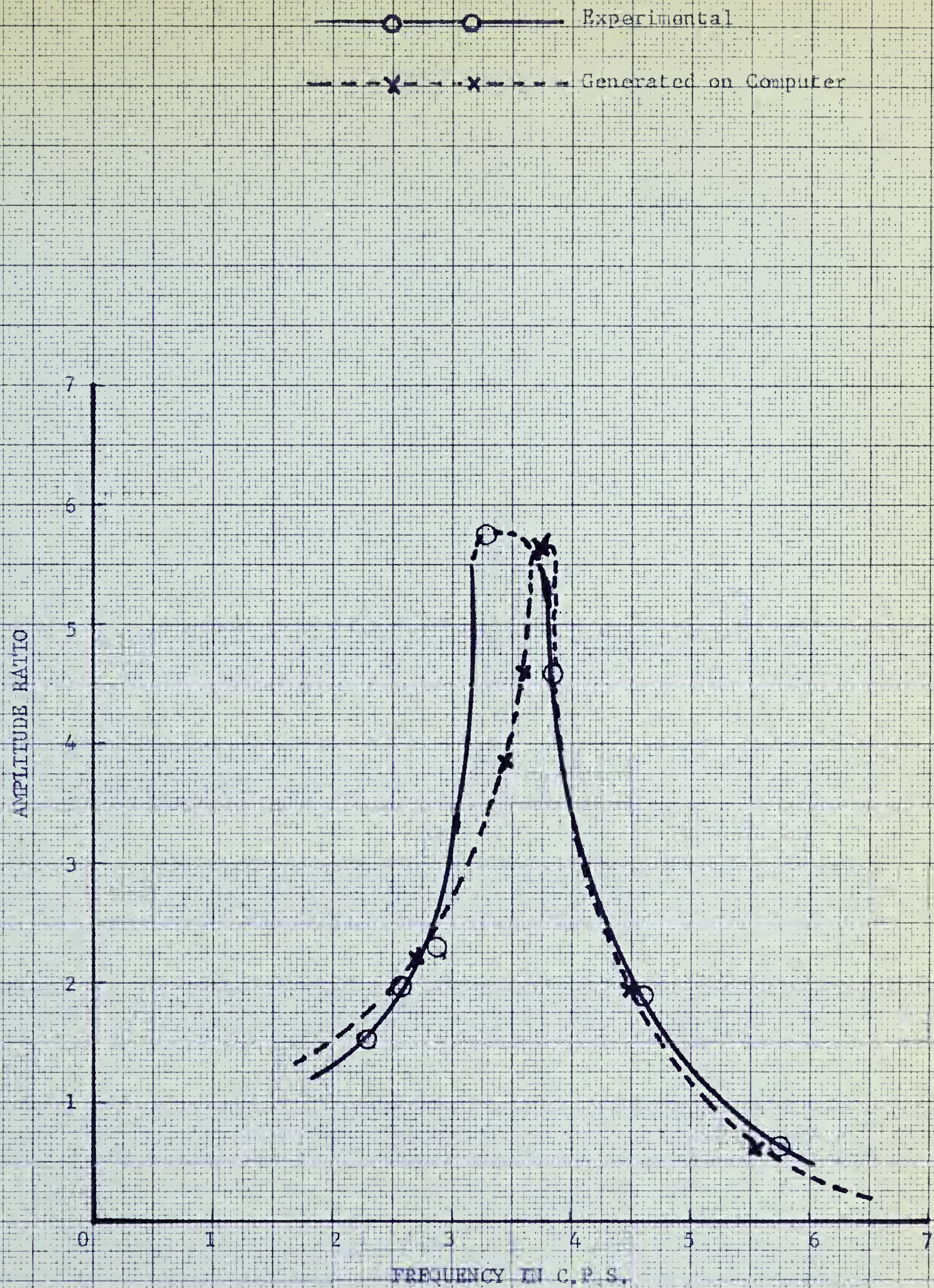
Figure 19: A Curve Obtained from the Computer Showing Amplitude Ratio and Damping.



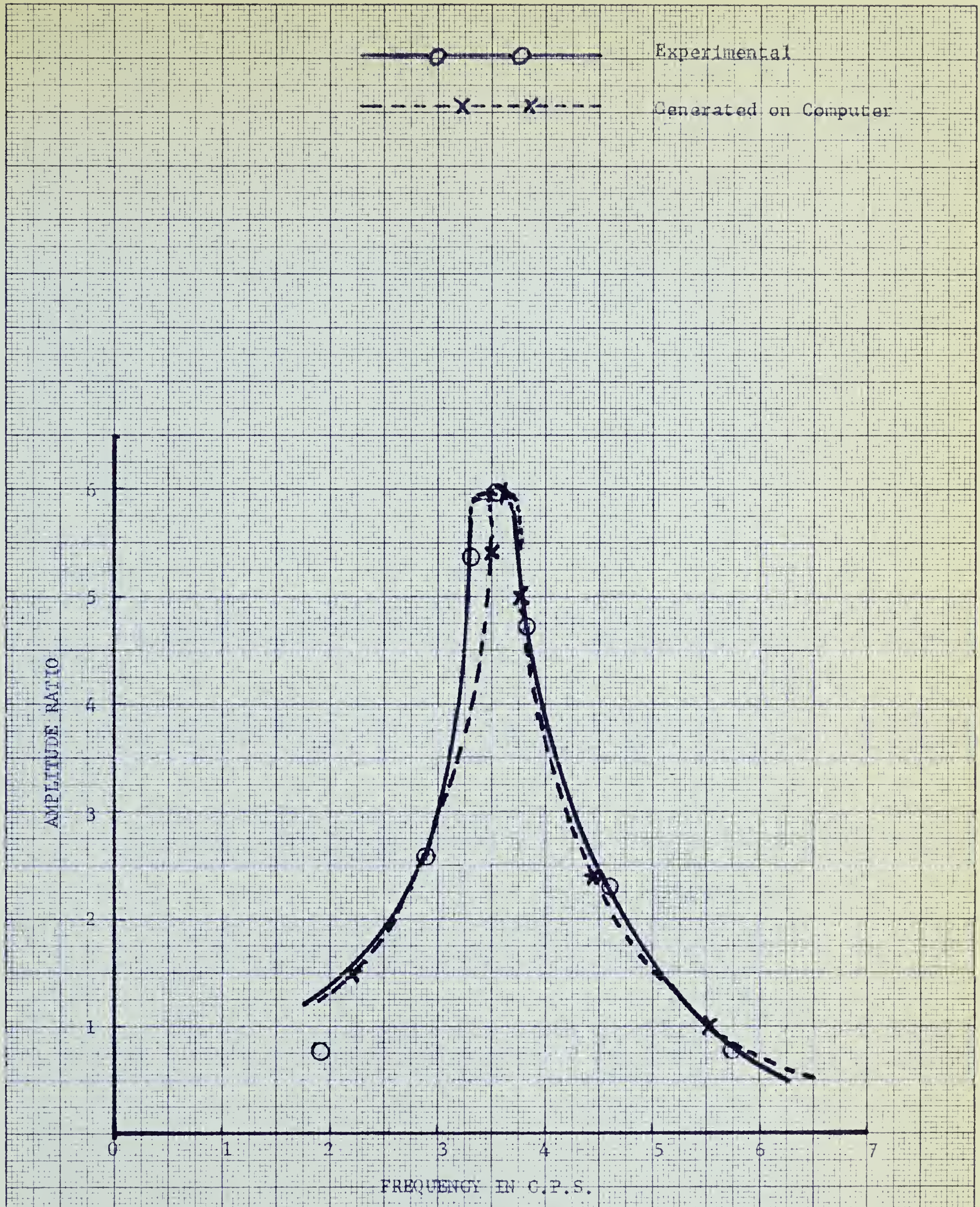
Graph 6: Amplitude Ratio Versus Frequency Curve for Front Tires. 0.145 inches Input.



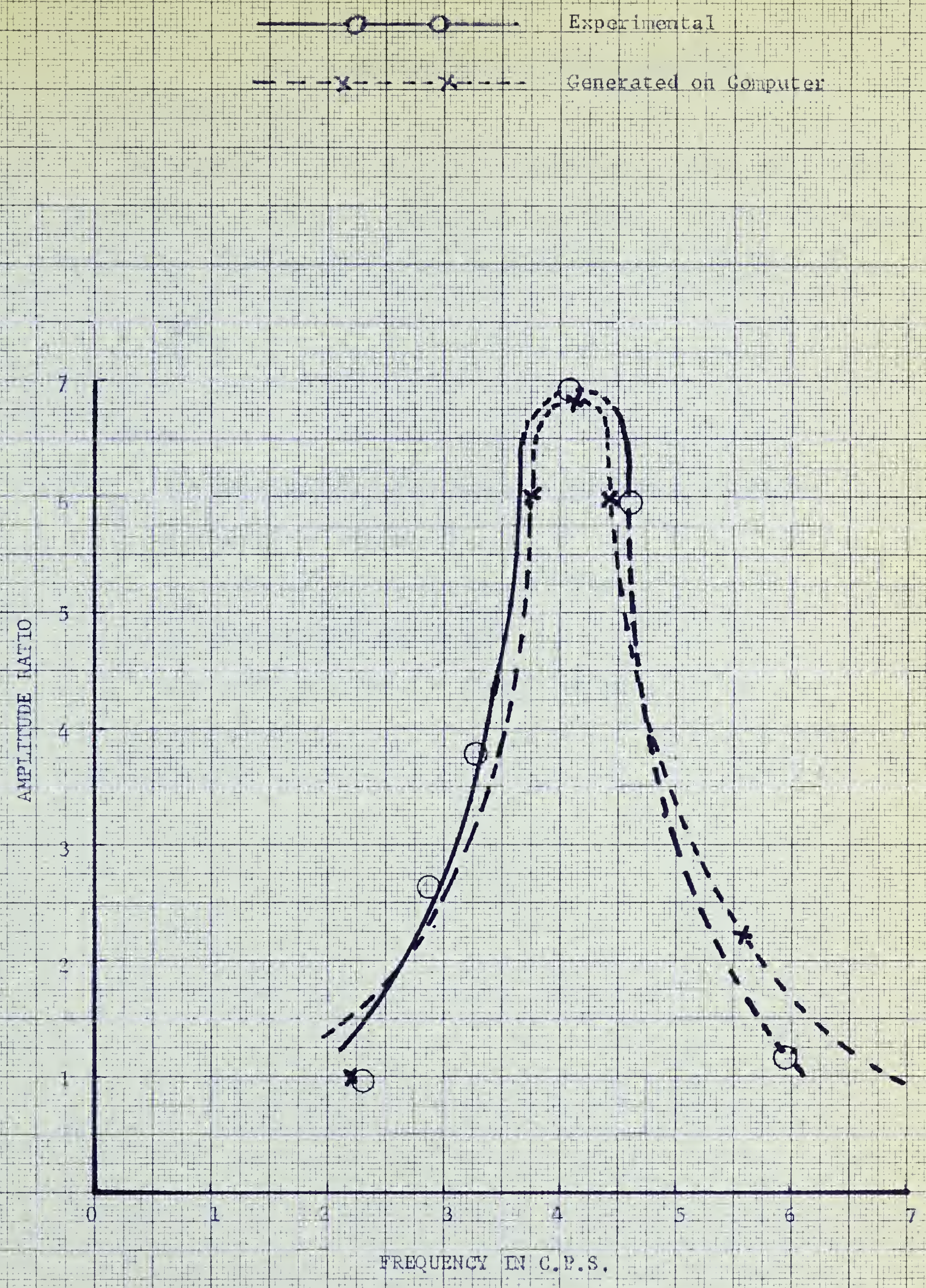
Graph 7: Amplitude Ratio Versus Frequency Curve for Front Wheel.
0.355 inches input.



Graph 8: Amplitude Ratio Versus Frequency Curve for 100% Air Filled Rear Tires.



Graph 9: Amplitude Ratio Versus Frequency Curve for 75% Liquid Filled Rear Tires.



Graph 10: Amplitude Ratio Versus Frequency Curve for Lead Ballast Rear Tires.

6. INVESTIGATION OF TRACTOR BEHAVIOUR

6.1 CENTRIFUGAL FORCE

Centrifugal force is one of the major forces which result in roll instability of a tractor. This force is present only when the tractor is turning. Equation 6 was used to calculate the centrifugal force.

$$F = \frac{Wv^2}{gr} \qquad (6)$$

Where

F = centrifugal force acting at the centre of gravity

W = weight of the tractor

v = velocity of the tractor in ft/sec

g = acceleration due to gravity

r = turning radius of a tractor in feet.

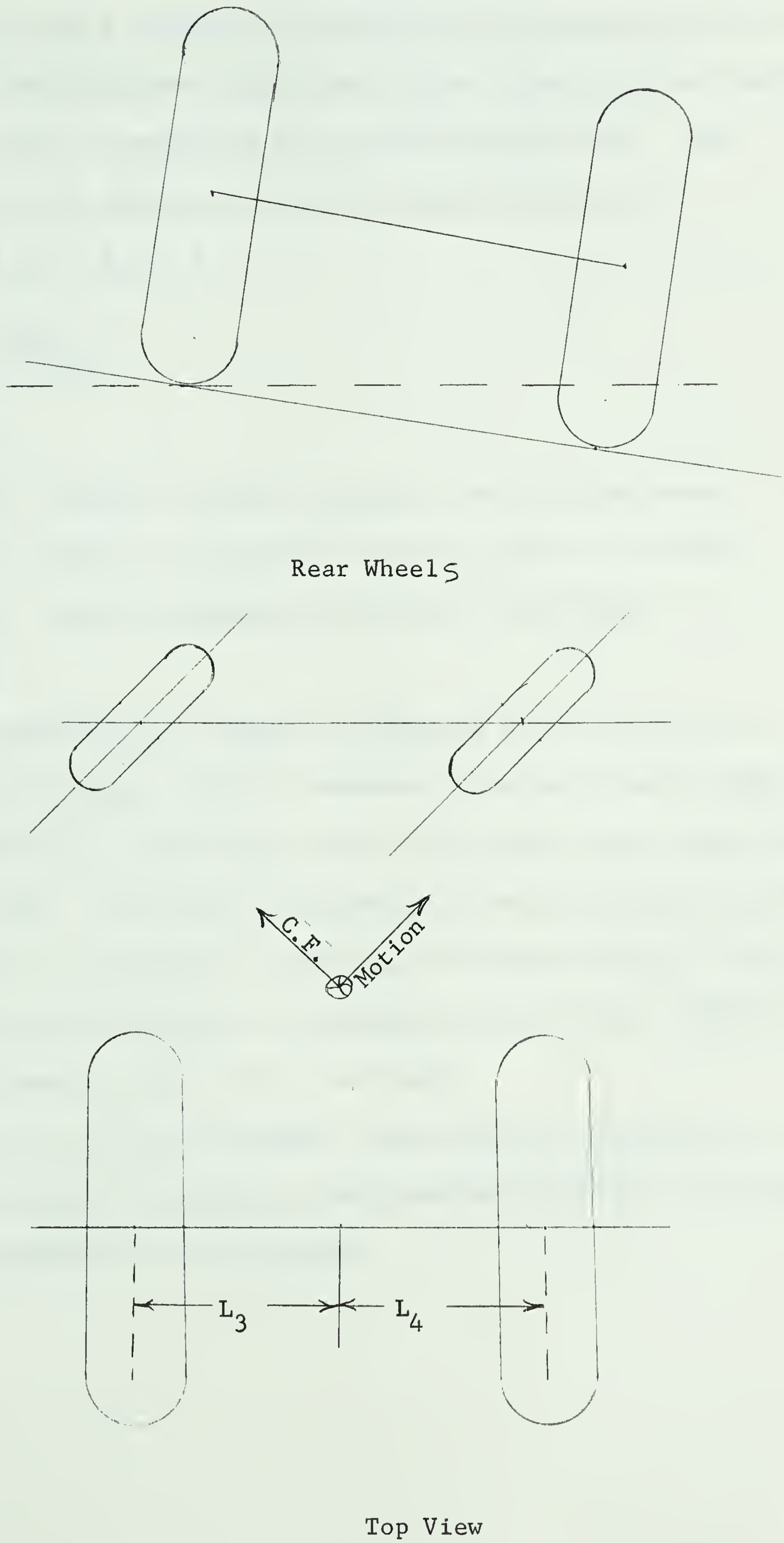


Figure 20: Centrifugal Force on a Tractor While Turning Along the Slope.

Roll stability of a tractor is affected by the vertical reactions on the wheels resulting from centrifugal force. These vertical reactions are always equal in magnitude but opposite in direction. The magnitudes of the reactions were determined by Equation 7.

$$Ft - F_1(L_3 + L_4) = 0 \quad \text{. . . . .} \quad (7)$$

$$F_1 = \frac{\pm Ft}{L_3 + L_4}$$

Where

F_1 = vertical reaction acting at the left rear wheel.

t = height of the centre of gravity above the roadway.

F_2 = vertical reaction on the right rear wheel.

6.2 SLOPE

Roll stability of a tractor is affected when the tractor is operating on a slope. In such conditions a "weight transfer" takes place. The wheel on the higher level will support less weight than the lower wheel. This effect of slope will help in stabilizing the tractor when it is taking a turn along the slope as shown in Figure 20.

If the tractor takes a turn opposite to the slope as shown in Figure 20 danger of roll over is increased.

A slope of 20° was simulated. The vertical components of centrifugal force were added to the transferred weight in supplying initial conditions for the computer.

6.3 CALCULATIONS FOR CENTRIFUGAL FORCE AND WEIGHT TRANSFER

6.3.1 Tractor Mounted on 100% Air Filled Tire. 12 psig

Weight of tractor with operator = 3565 lb.

Position of the centre of gravity without the operator,

29.2 inches forward from the centre line of the rear wheels.

Vertical distance above the roadway, 26.6 inches

New position of the centre of gravity with the operator,

28.4 inches forward from the centre line of the rear wheels.

Vertical distance above the roadway = 27.3 inches.

Wheel base = 72.0 inches.

Weight supported by the front wheels = $\frac{3565 \times 28.4}{72.0}$ = 1406 lbs.

Weight supported by the rear wheels = 2158 lbs.

Each rear wheel is supporting 1079 lbs.

6.3.1.1 Centrifugal Force

In calculating the centrifugal force combinations of 7.50, 15.0 mph speeds and 15.0 ft turning radius were considered.

Speed = 7.50 mph

Centrifugal force at a turning radius of 15.0 ft = $\frac{3565 \times 11.0 \times 11.0}{32.2 \times 15.0}$ = 893 lb.

Speed = 15.0 mph

Centrifugal force at a turning radius of 15.0 ft = $\frac{3565 \times 22.0 \times 22.0}{32.2 \times 15.0}$ = 3572 lb.

6.3.1.2 Vertical Reactions Due to Centrifugal Force.

In calculating the vertical components of the centrifugal force on each rear tire, a rear wheel tread of 48" was used.

$$F_{1-1} = \frac{F_1 t}{L_3 + L_4}$$

Where

$F_1 = 893$ lb.

$t = 27.3$ inches.

$L_3 + L_4 = 48.0$ inches.

$$F_{1-1} = \frac{893 \times 27.3}{48.0} = 507 \text{ lb.}$$

$$F_{1-2} = \frac{3570 \times 27.3}{48.0} = 2030 \text{ lb.}$$

Where

F_{1-1} = vertical reaction on the rear wheel due to centrifugal force at a speed of 7.50 mph and a turning radius of 15.0 ft.

F_{1-2} = the vertical reaction on any rear wheel due to the centrifugal force at a speed of 15.0 mph and a turning radius of 15.0 ft.

Vertical actions of centrifugal force will always act upward on the inner wheel and downward on the outer wheel. On level ground with 100% air filled tires at 15.0 mph speed, centrifugal force is sufficient to roll the tractor over at all turning radii of less than 25.0 feet.

6.3.1.3 The Tractor Operating on a Slope.

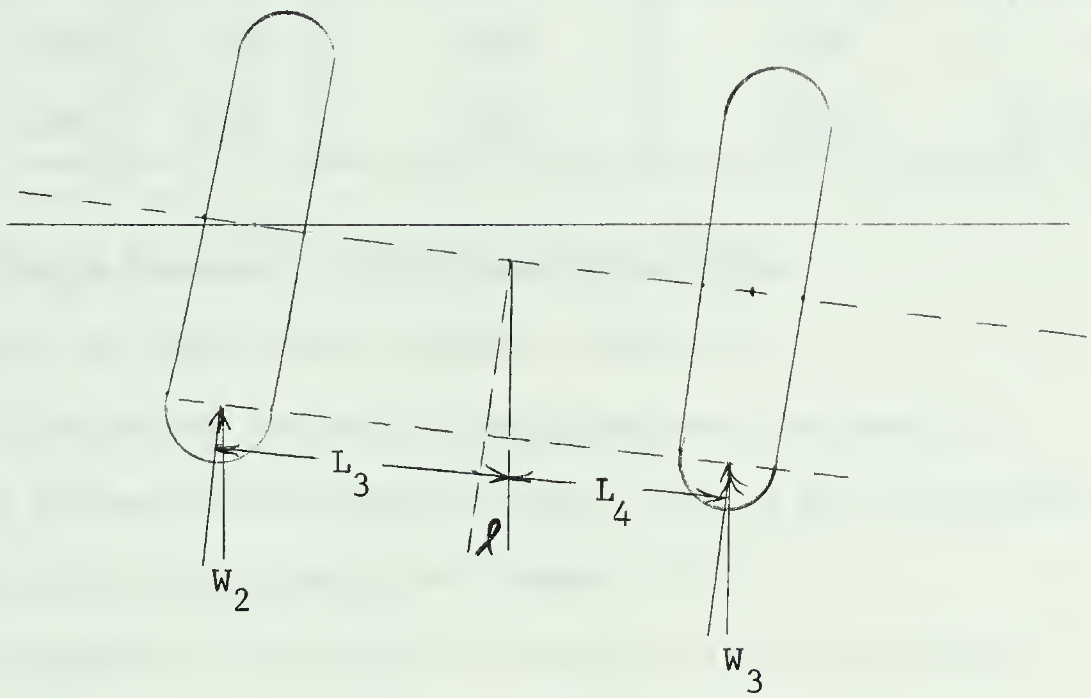


Figure 21: Weight Transfer Due to Slope

At 20° slope

$\ell = 27.3 \times \sin 20^{\circ}$

$= 27.3 \times 0.342$

$= 9.32''$

$L_3 = 33.3''$

$L_4 = 14.7''$

$W_3 = \frac{2158 \times 33.3}{48.0}$

$= 1498$ lb.

$W_2 = 660$ lb.

Weight supported by each rear wheel taking into consideration the effects of slope and centrifugal force is shown in the following table.

Table 18: Turning Across 20^{o} Slope (Tractor on 100% Air Filled Tire)

W_2*	W_3**	Speed	Vert. Action Due to C.F. on left rear tire	Vert. Action Due to C.F. on right rear tire	Net W_2	Net W_3
660	1498	7.50	-507	507	152	2006
660	1498	15.0	-2030	2030	-1369	3528

6.3.2 Tractor Mounted on 75% Liquid Filled Tire.

Weight of tractor with operator = 4405 lb.

Position of the centre of gravity without the operator,

23.4 inches forward from the centre line of the rear wheels.

Vertical distance above the roadway, 22.1"

New position of the centre of gravity with the operator,

22.9 inches forward from the centre line of the rear axle.

Vertical distance above the roadway = 22.9"

Wheel base = 72.0 inches

Weight supported by the front wheels = $\frac{4405 \times 22.9}{72.0}$ = 1403 lb.

Weight supported by the rear wheels = 3002 lb.

Each wheel is supporting 1501 lbs.

6.3.2.1 Centrifugal Force

In calculating the centrifugal force combinations of 7.50, 15.0 mph speed and 15.0 foot turning radius were considered.

Speed = 7.50 mph

Centrifugal force at a turning radius of 15.0 ft = $\frac{4405 \times 11.0 \times 11.0}{32.3 \times 15.0}$ = 1103.5 lb.

Speed = 15.0 mph

Centrifugal force at a turning radius of 15.0 ft = $\frac{4405 \times 22.0 \times 22.0}{32.2 \times 15.0}$ = 4414 lb.

* W_2 Weight supported by the left rear wheel.

** W_3 Weight supported by the right rear wheel.

6.3.2.2 Vertical Components of Centrifugal Force

Vertical components of the centrifugal force on each rear tire were as follows:

F_{1-1} = 527.61 lb.

F_{1-2} = 2110.44 lb.

On level ground with a 75% liquid filled tire at 15.0 mph speed the centrifugal force alone is sufficient to roll the tractor at all turning radii of up to 25.0 ft.

6.3.2.3 The Tractor Operating on a Slope.

A slope of 20^o was considered.

$\ell = 22.9 \times 0.342 = 7.84''$

$L_3 = 31.8''$

$L_4 = 16.2''$

$$W_3 = \frac{3002 \times 31.8}{48.0} = 1991 \text{ lb.}$$

W_2 = 1011 lb.

Weight supported by the left and rear wheels considering slope and centrifugal force is shown in the table that follows.

Table 19: Turning Across 20^o Slope. (Tractor on 75% liquid filled tire)

W_2*	W_3**	Speed	Vert. Action Due to C.F. on left rear tire	Vert. Action Due to C.F. on right rear tire	Net W_2	Net W_3
1011	1991	7.50	-527	527	483	2518
1011	1991	15.0	-2110	2110	-1099	4101

*W_2 Weight supported by the left rear wheel.

**W_3 Weight supported by the right rear wheel.

6.3.3 Tractor Mounted on Lead Ballast Tires.

Weight of tractor with operator = 4965 lb.

Position of the centre of gravity without the operator,

20.7 inches forward from the center of the rear axle.

Vertical distance above the roadway, 26.6"

New position of the center of gravity with the operator,

20.3 inches forward from the centre line of the rear wheels.

Vertical distance above the roadway = 27.2"

Wheel base = 72.0 inches

Weight supported by the front wheel $= \frac{4965 \times 20.3}{72.0} = 1403$ lb.

Weight supported by the rear wheels = 3562 lb.

Each wheel is supporting 1781 lb.

6.3.3.1 Centrifugal Force

In calculating the centrifugal force combinations of 7.50 and 15.0 mph speed and 15.0 foot turning radius were considered.

Speed 7.50 mph

Centrifugal force at a turning radius of 15.0 ft = 1243 lb.

Speed = 15.0 mph

Centrifugal force at a turning radius of 15.0 ft = 4975 lb.

6.3.3.2 Vertical Reactions Due to Centrifugal Force

Vertical components of the centrifugal force on each rear tire were as follows:

$F_{1-1} = 704$ lb.

$F_{1-2} = 2816$ lb.

On the level ground when the tractor is on the lead ballast tire it can take a safe turn at a 15.0 mph speed with a turning radius of 25.0 ft.

6.3.3.3 Tractor Operating on a Slope

A slope of 20^{o} was considered.

$\ell = 27.2 \times 0.342 = 9.29''$

$L_3 = 33.3''$

$L_4 = 14.7''$

$$W_3 = \frac{3562 \times 33.3}{48.0} = 2470 \text{ lb.}$$

$W_2 = 1092$ lb.

Weight supported by the left and rear wheels taking into consideration the effects of slope and centrifugal force is shown in the following table.

Table 20: Turning Across 20^{o} Slope (Tractor on lead ballast tire)

W_2*	W_3**	Speed	Vert. Action Due to C.F. on left rear tire	Vert. Action Due to C.F. on right rear tire	Net W_2	Net W_3
1092	2470	7.50	-704	704	388	3174
1092	2470	15.0	-2816	2816	-1724	5286

* W_2 Weight supported by the left rear wheel.

**W_3 Weight supported by the right rear wheel.

6.4 INITIAL CONDITIONS

The weight of the tractor gives initial deflection to the tractor tires. Weights supported by the left and right rear wheels and the centre of the front axle were calculated. After calculating these weights, vertical components of the centrifugal force acting at each rear tire were added algebraically to the initial weights. This provided the actual weight supported at each of the three points.

Knowing the spring rates of the tire and the weights supported by them made it possible to calculate the initial deflection. These deflections were simulated on the computer by initial voltages on the integrators. These initial voltages were changed each time for different speeds, turning radius and slopes.

Table 21 shows the initial conditions that were used on the computer

Table 21: Initial Conditions

Type of Ballast	Speed in mph	Run 1 level ground no turn	Run 2 level ground 15 ft turning radius	Run 3 20^o slope 15 ft turning radius	Run 4 20^o slope	X_1 in volts	X_2 in volts	X_3 in volts
100% Air Filled	7.5	1	-	-	-	0.11	0.602	0.602
"	"	-	2	-	-	"	0.32	0.884
"	"	-	-	3	-	"	0.251	1.12
"	"	-	-	-	4	"	0.368	0.835
75% Liquid Filled	"	5	-	-	-	"	0.677	0.677
"	"	-	6	-	-	"	0.439	0.915
"	"	-	-	7	-	"	0.238	1.136
"	"	-	-	-	8	"	0.455	0.898
Lead Ballast Tire	"	9	-	-	-	"	0.476	0.476
"	"	-	10	-	-	"	0.288	0.6645
"	"	-	-	11	-	"	0.1037	0.848
"	"	-	-	-	12	"	0.292	0.66
100% Air Filled	15	13	-	-	-	"	0.602	0.602
"	"	-	-	-	14	"	0.368	0.835
75% Liquid Filled	"	15	-	-	-	"	0.677	0.677
"	"	-	-	-	16	"	0.455	0.898
Lead Ballast Tire	"	17	-	-	-	"	0.476	0.476
"	"	-	-	-	18	"	0.292	0.66

7. RESULTS AND DISCUSSION

7.1 EXPERIMENTAL RESULTS

The amplitude ratio at any frequency varied with the input system for the same tire ballast. For example, in the case of 100% air filled tires amplitude ratio was 1.66 (Table 2) with the eccentric roller speed of 92 rpm while with the other input system, the amplitude ratio was only 1.53 (Table 7) at an input frequency of 138 cpm. Similar results were obtained in the case of 75% liquid filled and lead ballast tires by comparing Table 3 to Table 8 and Table 4 to Table 9. This is probably due to the centrifugal force of the tractor tires when driving the roller in the first system (Figure 6). By observing Graph Numbers 1 and 3 it is clear that when the tractor is driving the roller (Figure 6) amplitude ratio versus frequency curve of the 75% liquid filled ballast is close to the amplitude ratio versus frequency of the lead ballast tire, but when the other input system was used, (Figure 7) the amplitude ratio versus frequency of the 75% liquid filled ballast is close to the amplitude ratio versus frequency curve of the 100% air filled tires. This is also probably due to centrifugal force of the tractor tire when driving the roller (Figure 6). The results of tests with both input systems indicated that at the natural frequency, lead ballast tires bounce more than liquid filled and air filled tires. At natural frequencies, the amplitude ratio of lead ballast tires was 6.9 (Table 9), for 75% liquid filled tires it was 5.96 (Table 8) and for 100% air filled tires it was 5.74 (Table 7). This relationship between amplitude ratios at the natural frequencies may be explained by the difference in tractor weights.

The output amplitude in a forced vibration single degree of freedom system is given by the equation:

$$X = \frac{Fo}{\left((k - mw^2)^2 + (cw)^2\right)^{\frac{1}{2}}}$$

Where

X = amplitude of steady oscillation

Fo = force

k = spring rate

m = mass

w = frequency

c = damping factor

This theory shows that as mass increases, the output amplitude also increases for the same force. Results of the tests supported the theory.

The three types of tires had three different natural frequencies. Air filled tires had the minimum natural frequency, 3.28 cps (Table 7), 75% liquid filled tires had a natural frequency of 3.55 cps (Table 8) and lead ballast tires had a natural frequency of 4.07 cps (Table 9). At any frequency beyond the natural frequency, the amplitude ratio of the lead ballast tire was higher than the amplitude ratios for the 75% liquid filled and 100% air filled tires. Similarly at any frequency beyond the natural frequency, 75% liquid filled tires had a higher amplitude ratio than 100% air filled tires.

In running the actual programme coefficients K_6, K_{13} and K_{17} were changed.

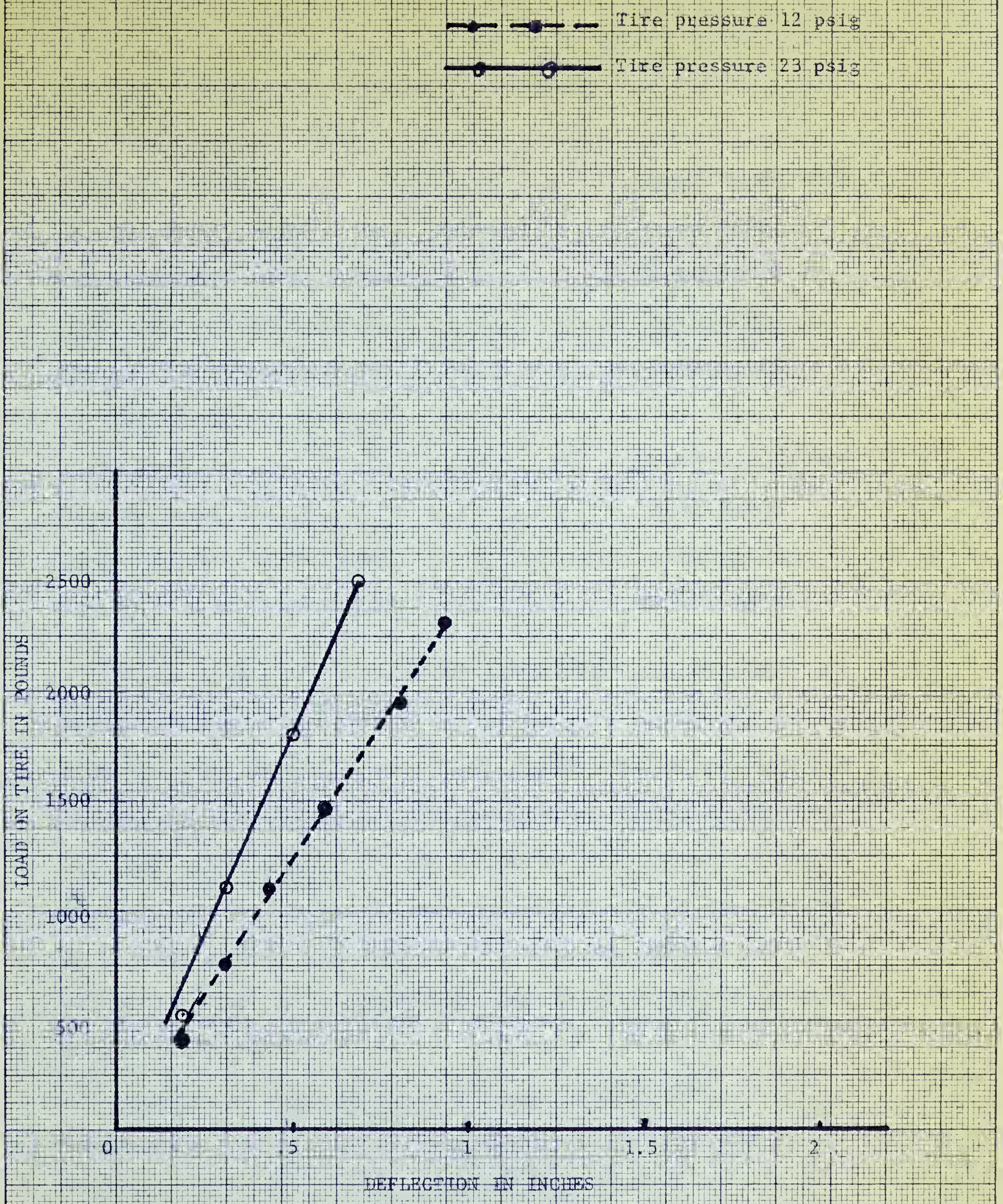
$$K_6 = \frac{C_2}{mk_1}$$

$$K_{13} = (L_1 + L_2) \frac{L_2 C_2}{I\Theta k_3}$$

$$K_{17} = \frac{L_4 C_3}{I\Phi} (L_3 + L_4)$$

In the case of 100% air filled tires the above coefficients were increased 30% but in the case of 75% liquid filled and lead ballast tires they were increased 50% and 60% respectively. One of the reasons might have been that frequency versus amplitude ratio curves were obtained with a single degree of freedom system, but the actual program was run as three degree of freedom system. There were chances of mass moment of inertia effecting the system also.

To determine the degree of nonlinearity of the system two input amplitudes were used. Input one was 0.1455 inches peak to peak and gave a natural frequency of 4.6 cps (Table 5). Input two was 0.355 inches peak to peak and gave a natural frequency of 4.18 cps (Table 6). The maximum frequency shift from the low values obtained for 0.355 inch peak to peak to high values obtained for 0.1455 inch peak to peak was about 10 percent higher. Raney[14] observed some nonlinearity, since with a change of input amplitude there was a frequency shift. The test indicated that there is some non linearity present in the system, but its overall effect is not great. It can be treated as a linear system as was treated in this project.



Graph 11: Comparison of Load Versus Deflection Curves for Tire Pressures of 12 psig and 23 psig.

Static spring rate tests gave the following results.

Tire pressure - 23 psig

Condition of test	Static Spring Rate
Tire on ground (level)	21,818 lb/ft
Tire on 6" x 6" wooden block	10,100 lb/ft
Tire on 6" x 4" wooden block	9,250 lb/ft
Tire on 6" x 2" wooden block	7,800 lb/ft

Tire pressure - 12 psig

Condition of test	Static Spring Rate
Tire on ground (level)	15,085 lb/ft
Tire on 6" x 6" wooden block	4,650 lb/ft
Tire on 6" x 4" wooden block	3,925 lb/ft

Static spring test results led to the conclusion that the tire spring rate varies with bump size. In fact, the spring rate is different when the tire first contacts a bump, when it is partially on the bump, and when it is fully on the bump. Static spring rate (on level ground) was 1.45 times greater at 23 psig pressure than at 12 psig pressure. But at the same pressures static spring rate was 2.175 times greater than when the tires were resting on 6" x 6" wooden blocks and 2.36 times greater when the tires were resting on 6" x 4" wooden blocks.

In the author's opinion conducting more experiments of this nature might lead to a formula expressing static spring rate in relation to tire type, air pressure and bump size.

7.2 ANALOG COMPUTER SOLUTIONS

Graphs obtained on the computer are shown in Appendix I. Displacements of the left rear wheel and right rear wheel were recorded directly without any multiplication factor. The output of the centre of the front axle was multiplied by five before recording on the recorder. This was done because the displacement of the front axle centre was considerably less than the output of the rear wheel centres.

7.2.1 Dynamic Spring Rate and Damping Coefficients.

To obtain values for the dynamic spring rate and damping, system equations were simulated on an electronic analog computer as indicated on Pages 55-65. Computer simulation yielded the following values for the dynamic spring rate and damping.

Table 22: Dynamic Spring Rate and Damping of Tractor Tires.

Tire	Air pressure in psig	Dynamic Spring Rate in lbs/ft	Damping in lb.sec/ft
Front tire 100% air filled	26.00	38,465.00	121.00
Rear tire 100% air filled	12.00	5,376.00	250.00
Rear tire 75% liquid filled	12.00	6,648.00	331.00
Rear tire Lead ballast	-	11,217.00	465.00

The dynamic spring rates obtained from the computer solutions appear to be low by comparison with Raney[13]. This might have been due to the limited area of contact between the tires and the platform, as shown in Figure 7.

7.2.2 Tractor Stability

The mathematical model, expressing the tractor attitude in terms of the vertical position of three suspension points, did not permit a simulated overturn. The model did however permit a determination of the points at which tires bounced clear of the ground. The number of times wheels left the ground was considered to be a true indication of relative stability. Results are therefore shown as the number of times each suspension point cleared the ground. The graphs, recorded in Appendix I were analyzed using this criterion. The results are tabulated below.

Table 23: Tractor on 100% air filled tire - Operating Speed 7.5 mph

Run No.	Number of times left rear wheel left the ground in 1320 feet of travel.	Number of times right rear wheel left the ground in 1320 feet of travel.
1. Level ground No Turn	35	40
2. Level ground 15' Turning Radius	62	12
3. 20^{o} slope 15' Turning Radius	70	13
4. 20^{o} slope No Turn	60	16

Table 24: Tractor on 75% liquid filled tire - Operating speed 7.5 mph

Run No.	Number of times left rear wheel left the ground in 1320 feet of travel.	Number of times right rear wheel left the ground in in 1320 feet of travel.
1. Level ground No Turn	33	34
2. Level ground 15' Turning Radius	51	19
3. 20^{o} slope 15' Turning Radius	60	6
4. 20^{o} slope No Turn	45	15

Table 25: Tractor on lead ballast tire - Operating speed 7.5 mph

Run No.	Number of times left rear wheel left the ground in 1320 feet of travel.	Number of times right rear wheel left the ground in 1320 feet of travel.
1. Level ground No Turn	25	23
2. Level ground 15' Turning Radius	60	20
3. 20^{o} slope 15' Turning Radius	103	7
4. 20^{o} slope No Turn	75	30

Table 26: Comparison of Stability at an Operating Speed of 15 mph.

Type of Ballast	Run No.	Number of times left rear wheel left the ground in 1320 feet of travel.	Number of times right rear wheel left the ground in 1320 feet of travel.
100% air filled	1. Level ground No Turn	30	25
	4. 20^{o} slope No Turn	26	15
75% liquid filled	1. Level ground No Turn	20	15
	4. 20^{o} slope No Turn	35	11
Lead ballast	1. Level ground No turn	16	18
	4. 20^{o} slope	40	8

Table 27: Analysis of Graph for Front Axle Centre.

Type of Ballast	Speed in mph	Number of times front wheel left the ground
100% air filled	7.5	29
	15	13
75% liquid filled	7.5	35
	15	25
Lead ballast	7.5	60
	15	45

A comparison of Tables 23, 24, 25 and 26 leads to the conclusion that the tractor tires left the ground more frequently when being operated at a speed of 7.50 mph than when operating at a speed of 15.0 mph. This confirms the observations of Dahir and Stout[5].

75% liquid filled ballast tires left the ground fewer times than 100% air filled tires. This does not agree with Delzel[6].

The lead ballast tires left the ground fewer times when operating on level ground with no turns, but they left the ground more frequently than the 100% air filled and 75% liquid filled tires when operating on sloping ground or turning.

Longitudinal instability was most acute in the case of the lead ballast tires and least in the case of the 100% air filled tires. (Table 27). Longitudinal instability became less acute as speed increased (Table 27).

7.3 CONCLUSIONS

Experiments with a tractor subject to sinusoidal displacements to the rubber tired rear wheels, yielded values for natural frequencies as follows:

Air filled - 3.28 cps
75% liquid filled - 3.55 cps
Lead ballasted - 4.07 cps

Dynamic spring rates and damping coefficients were found to be:

Air filled rear tires, 12 psi, 5376.0 lb/ft and 325 lb.sec/ft.
75% liquid filled tires, 12 psi, 6648.0 lb/ft and 496 lb.sec/ft
Lead ballasted rear tires, 12 psi, 11217.00 lb/ft and 740 lb.sec/ft
Front tires, 26 psi, 38,465.0 lb/ft and 121.0 lb/sec/ft

For all ballasts, both roll and pitch stability were less at 7.5 mph than at 15 mph. Roll stability on sloping ground while turning, was highest for 75 % liquid filled tires.

For all ballasts, tires were far from being critically damped. Stability therefore could be increased by development of a ballast that would increase tire damping.

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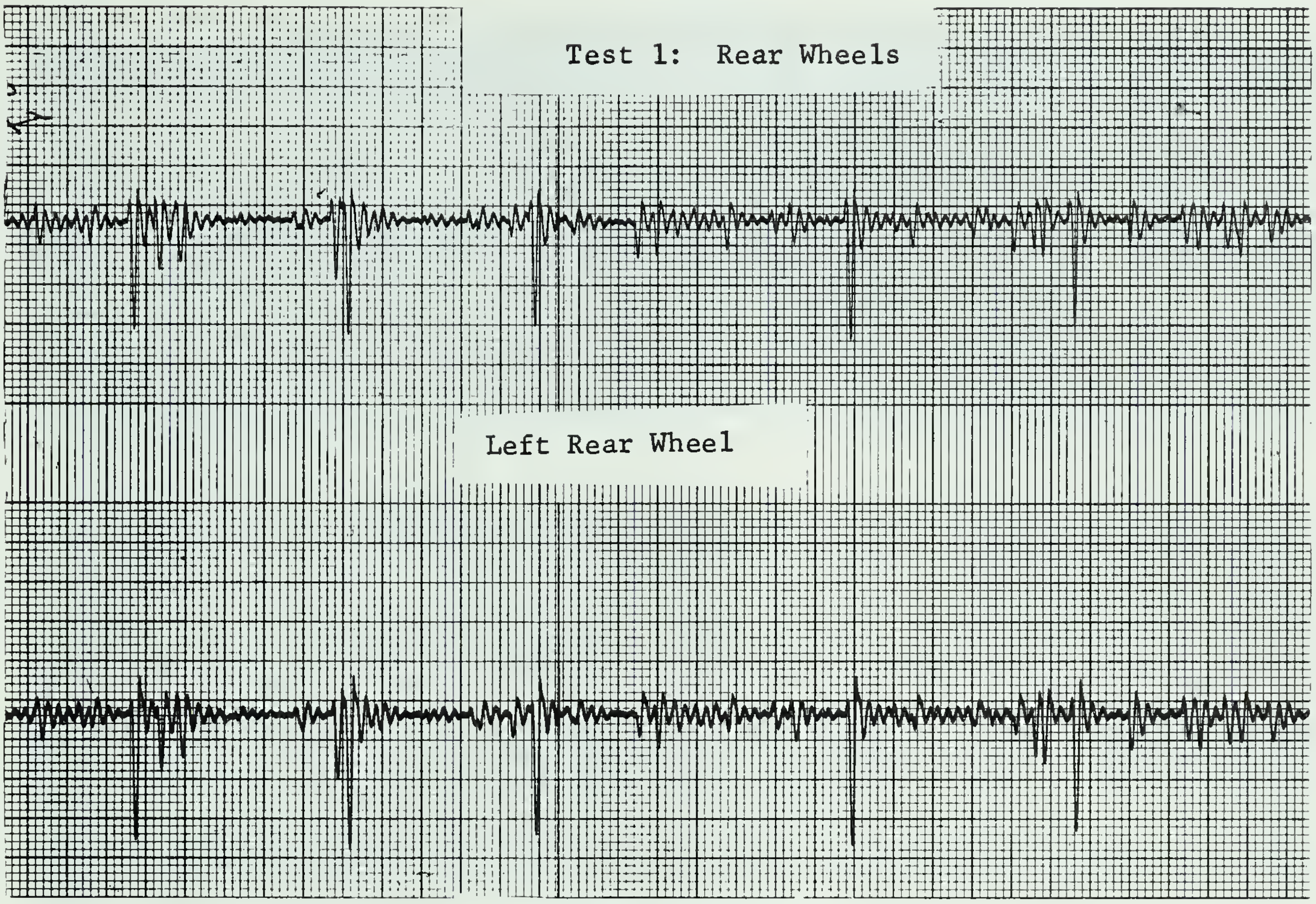
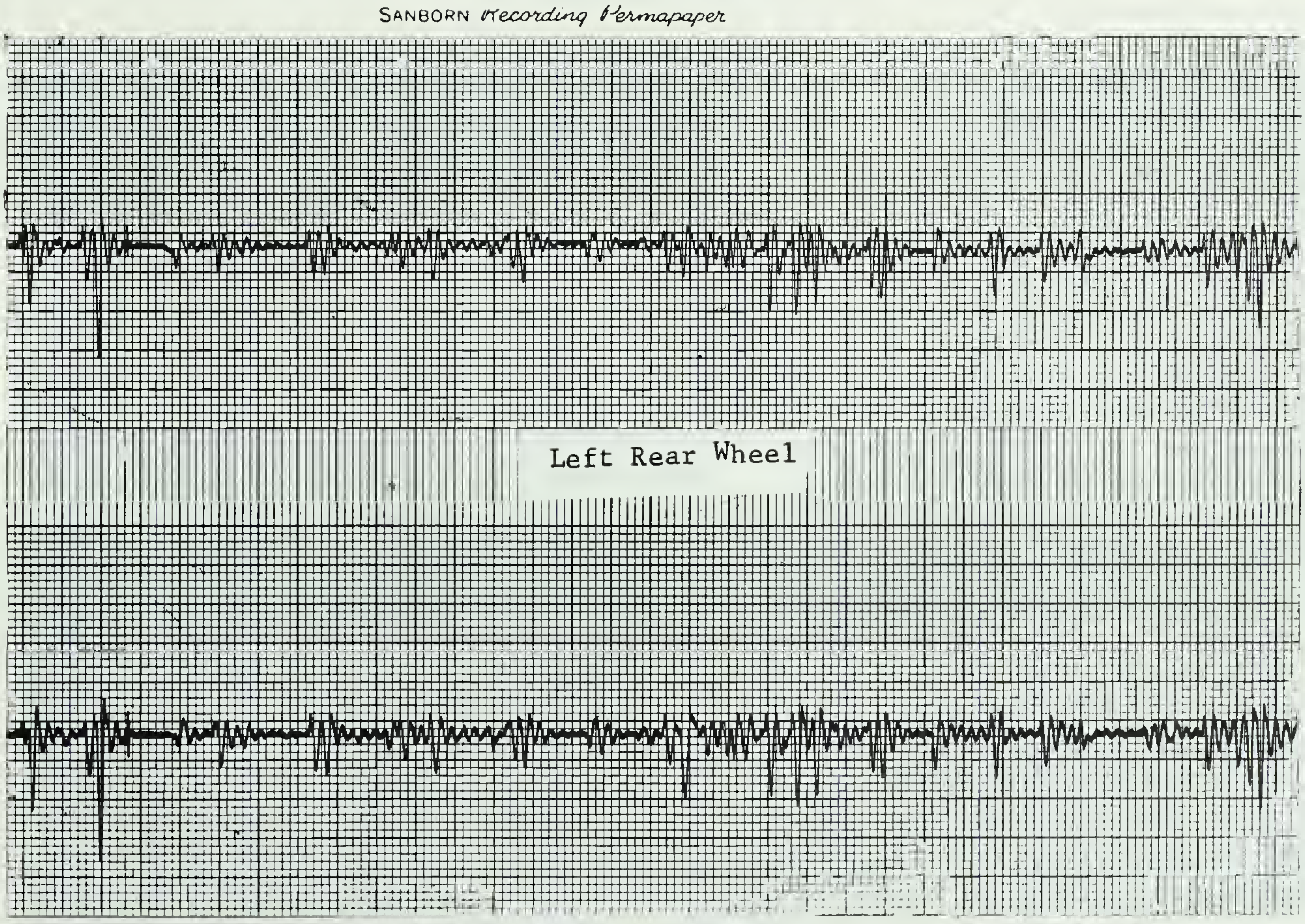
APPENDIX



Right Rear Wheel

SANBORN Recording Permapaper



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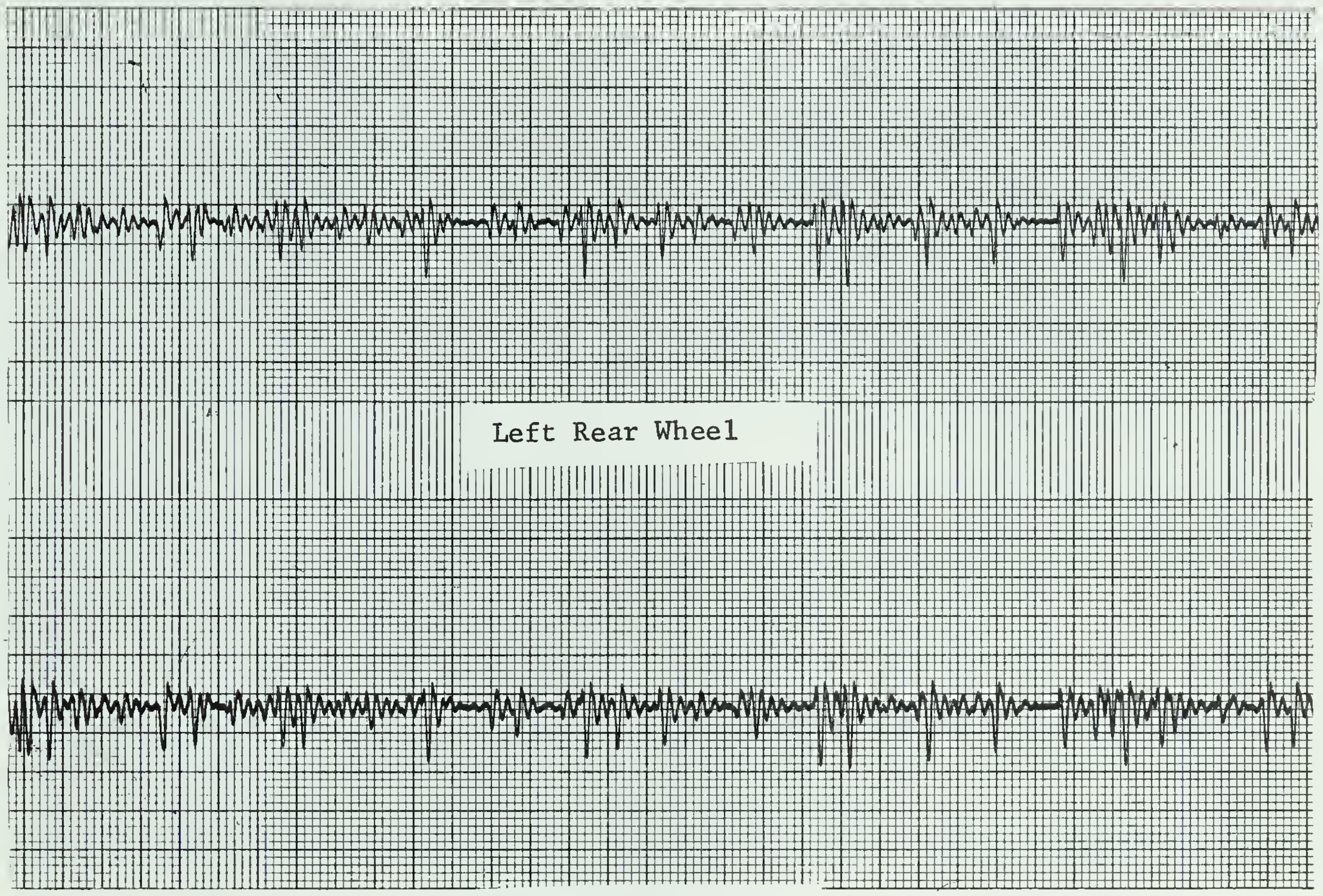

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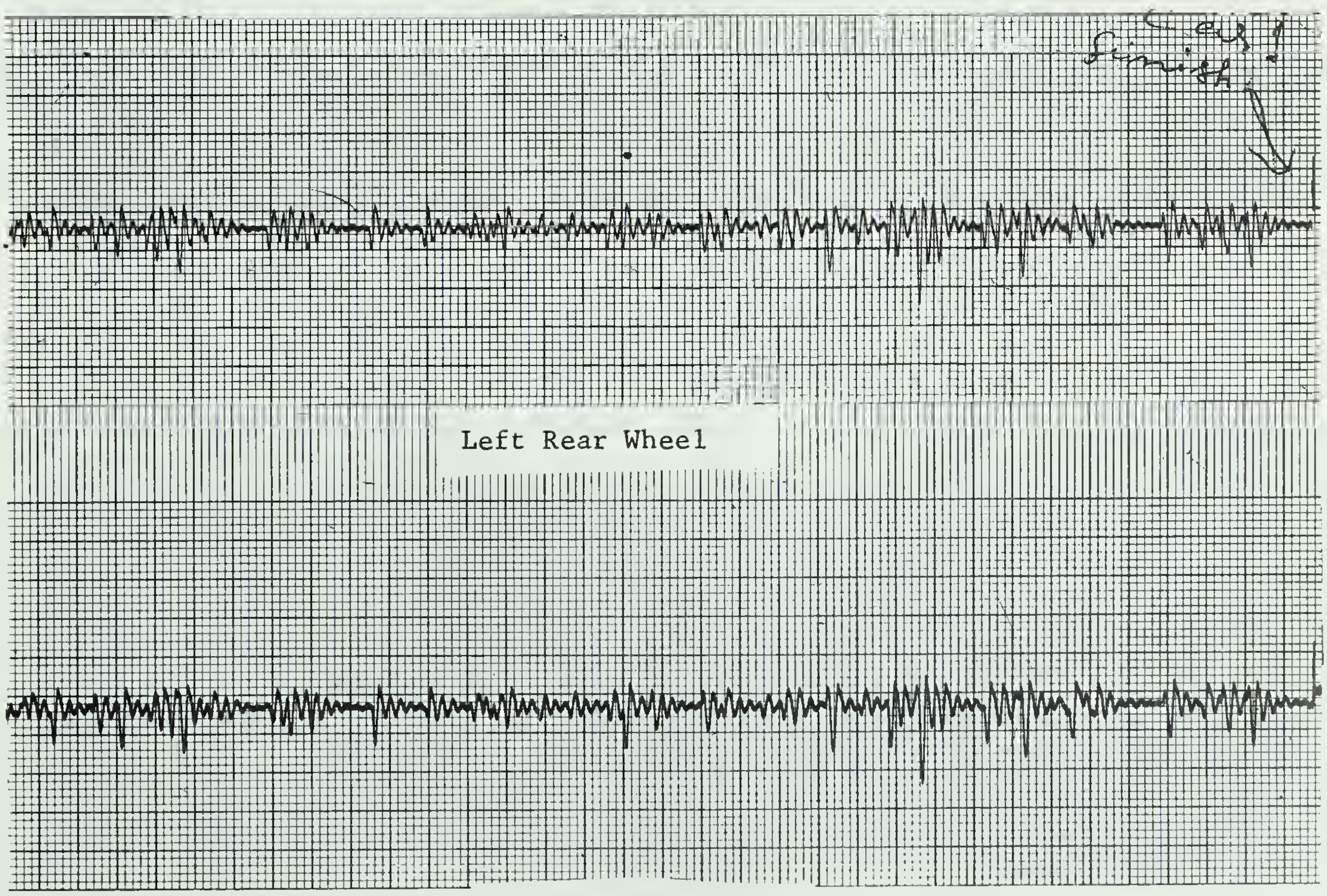

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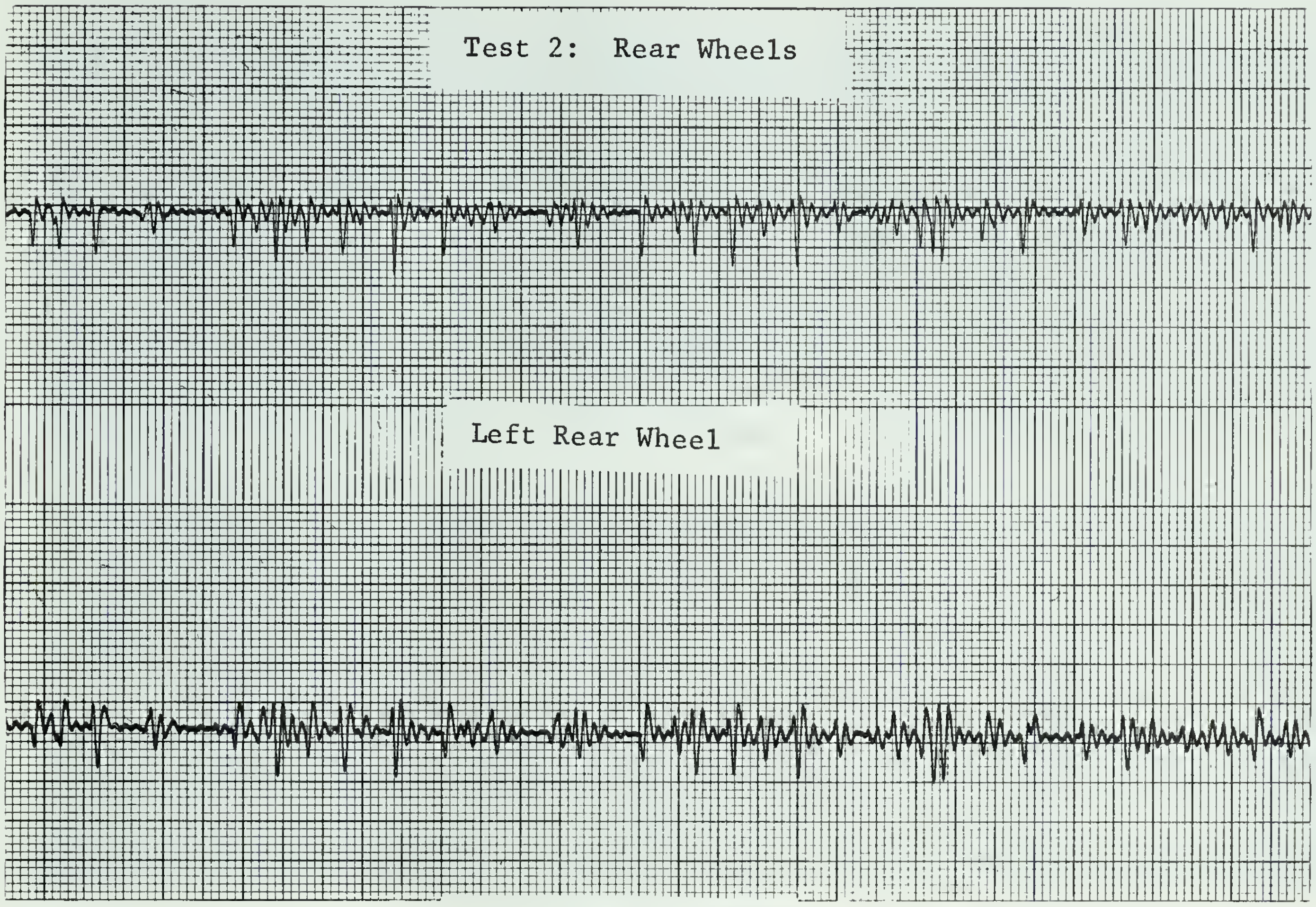

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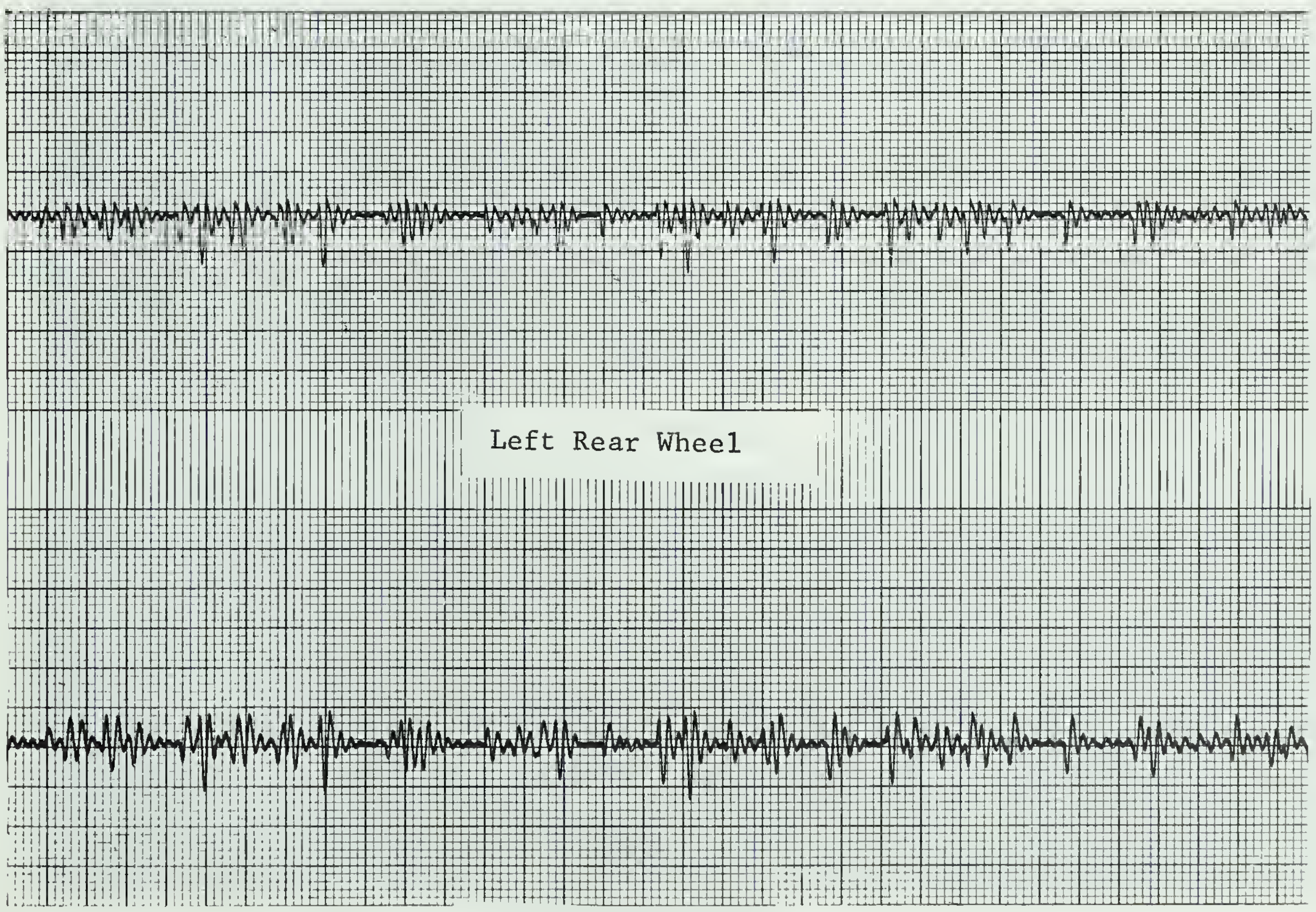
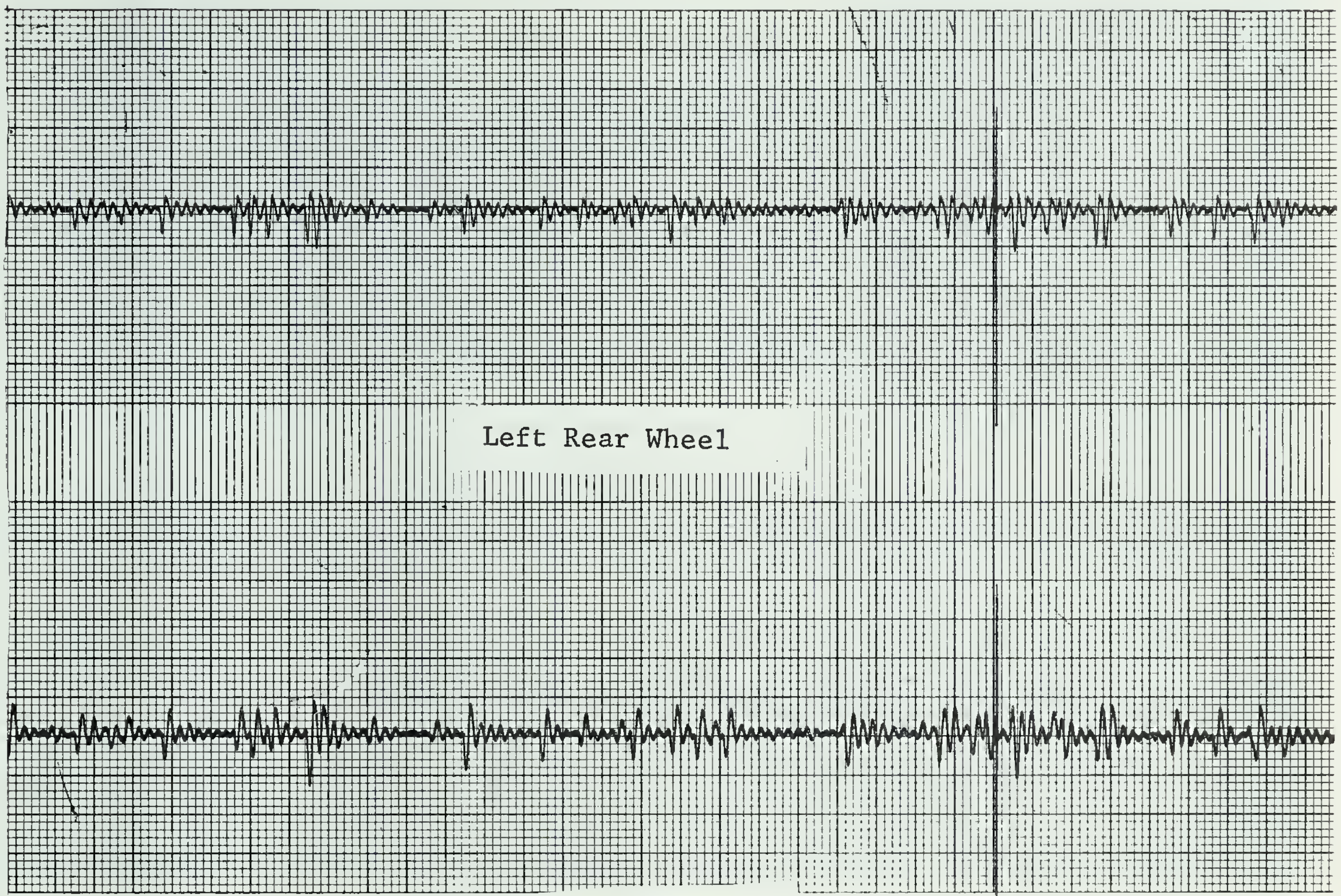


Right Rear Wheel



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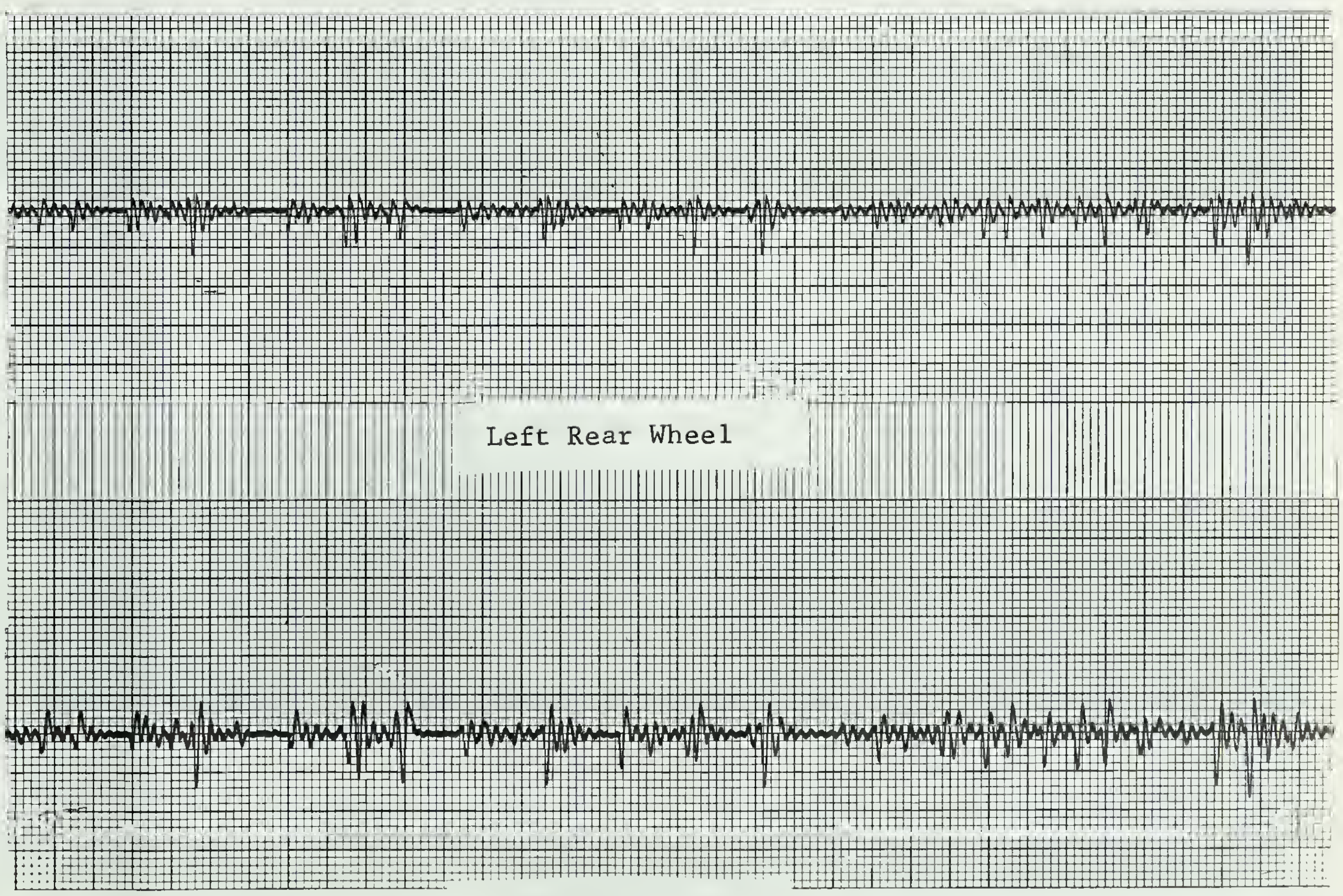

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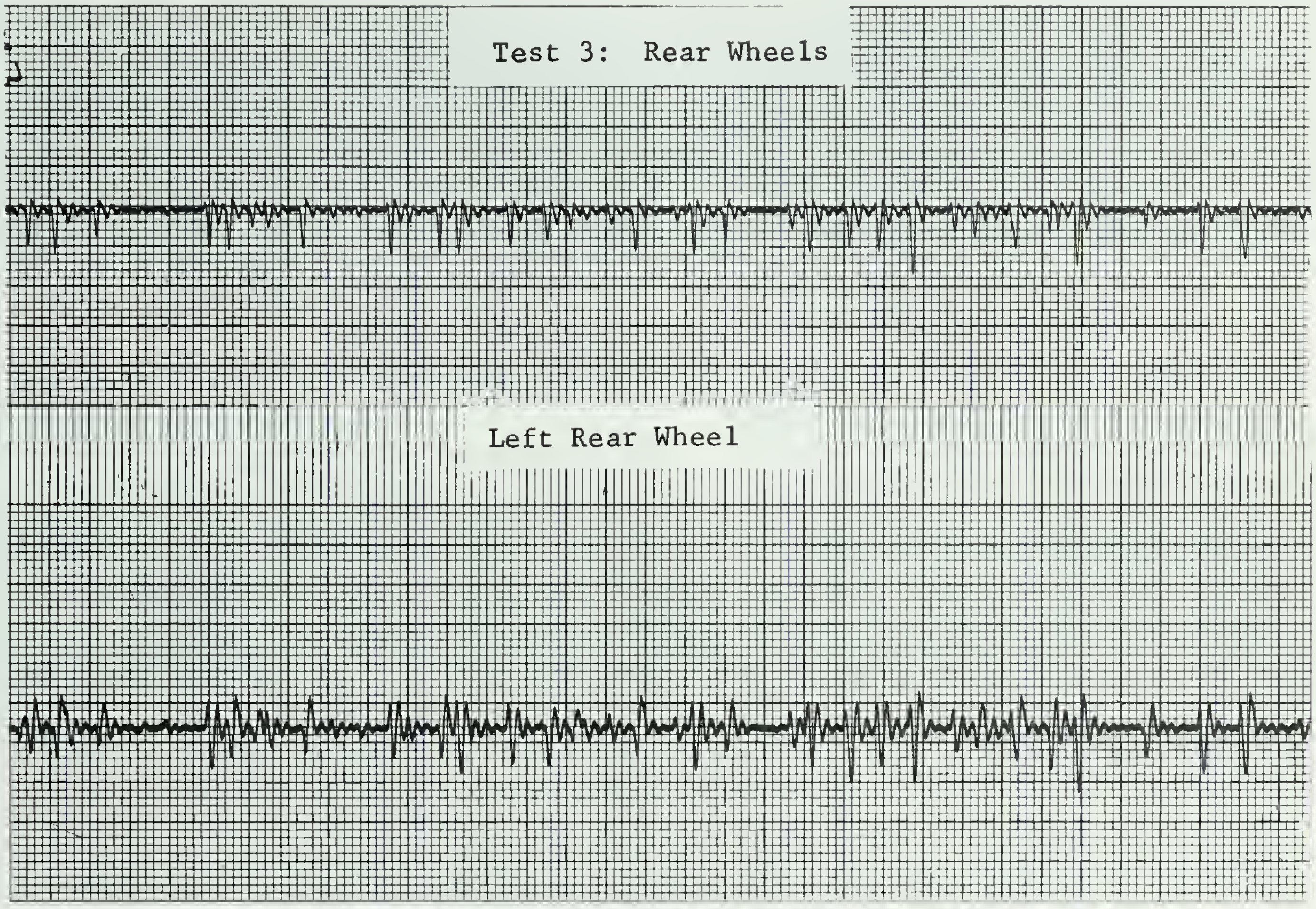

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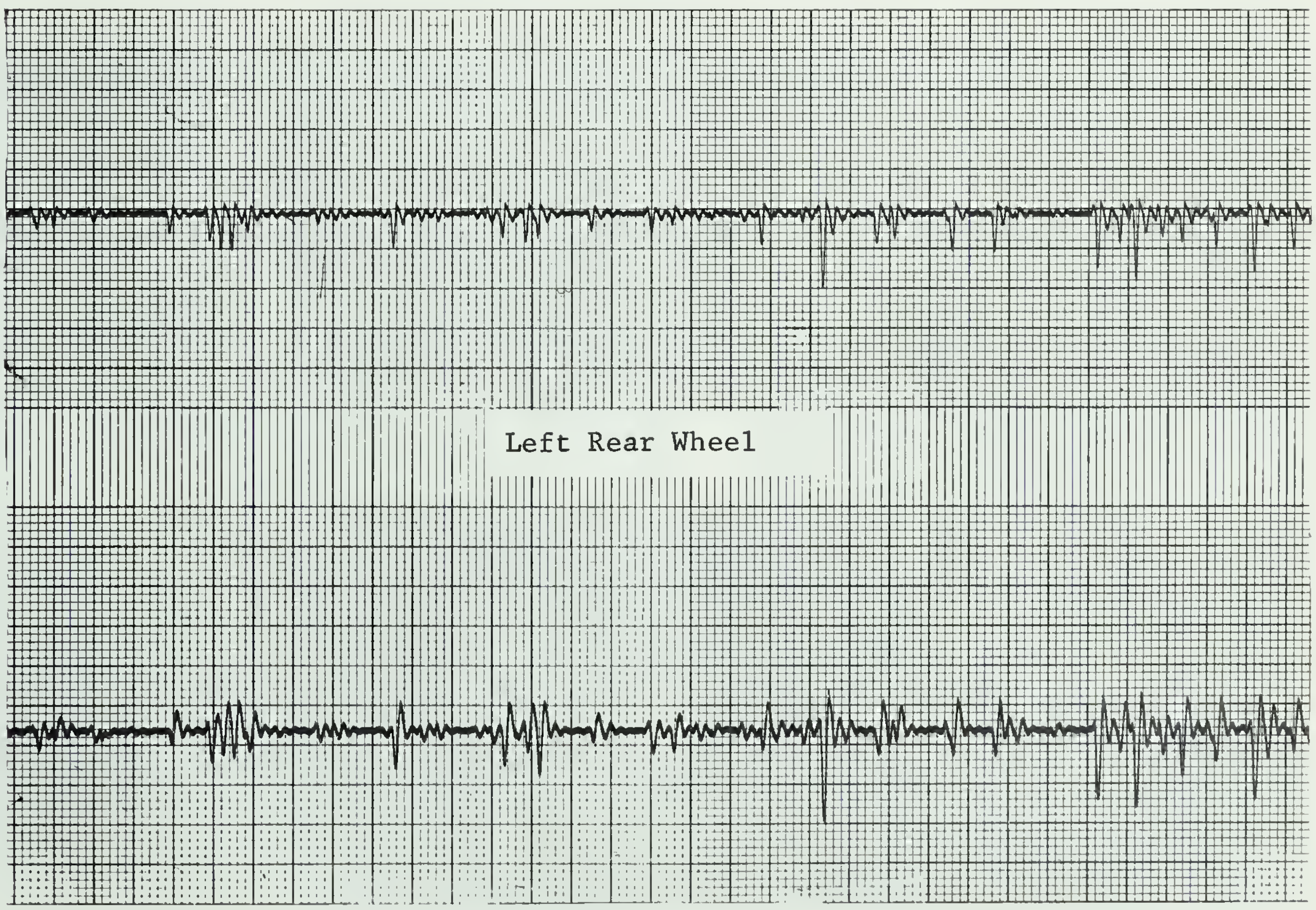

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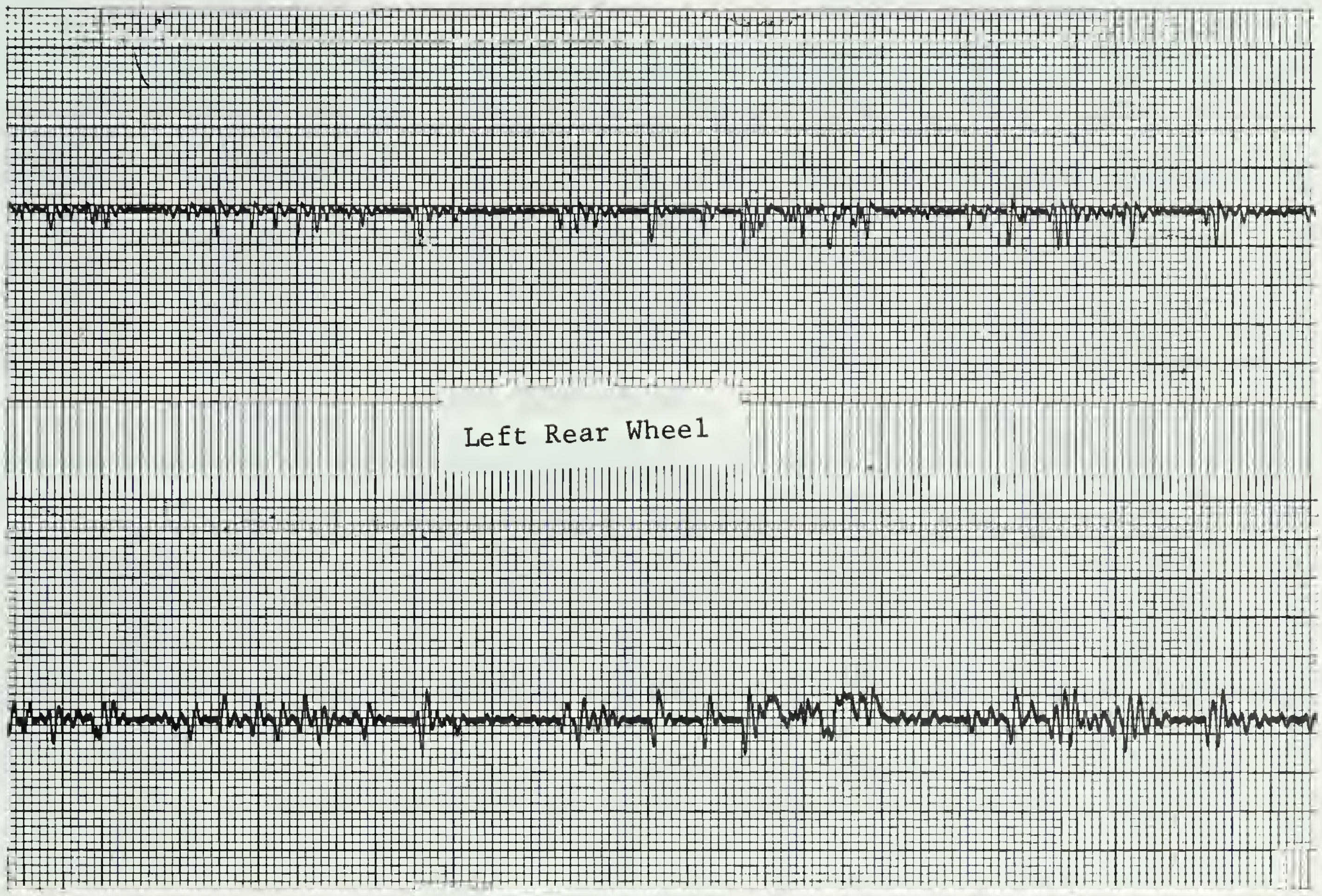

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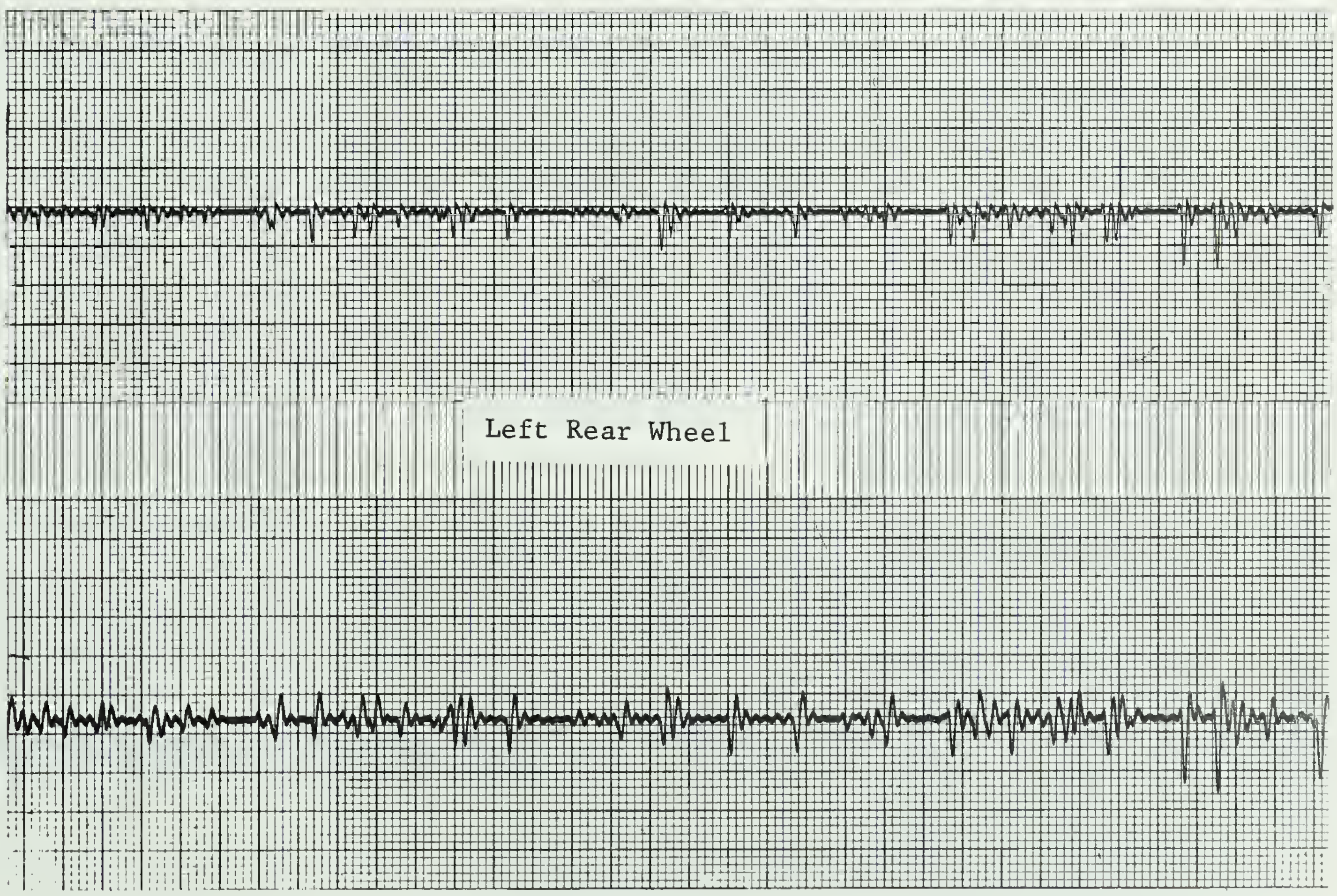

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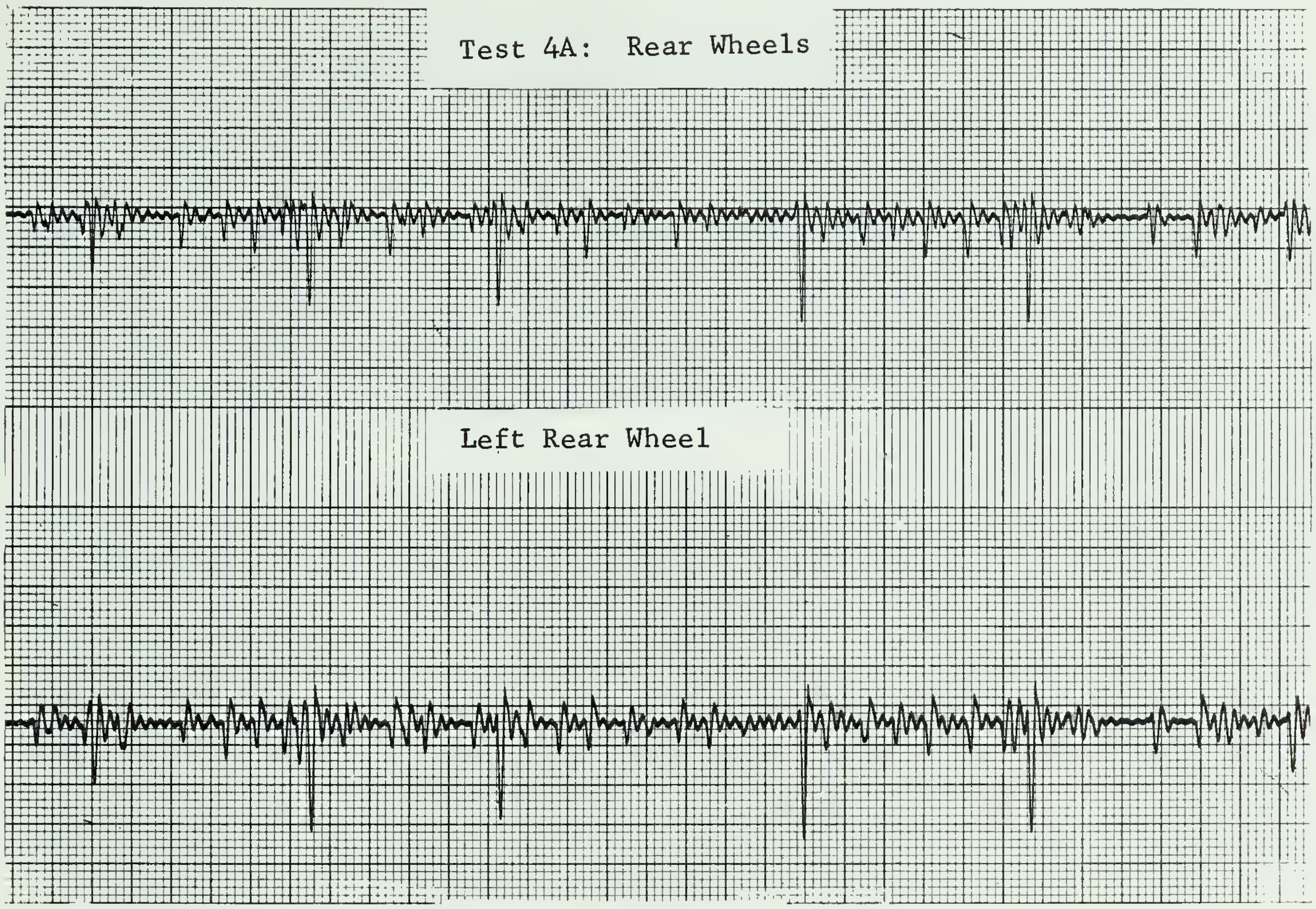

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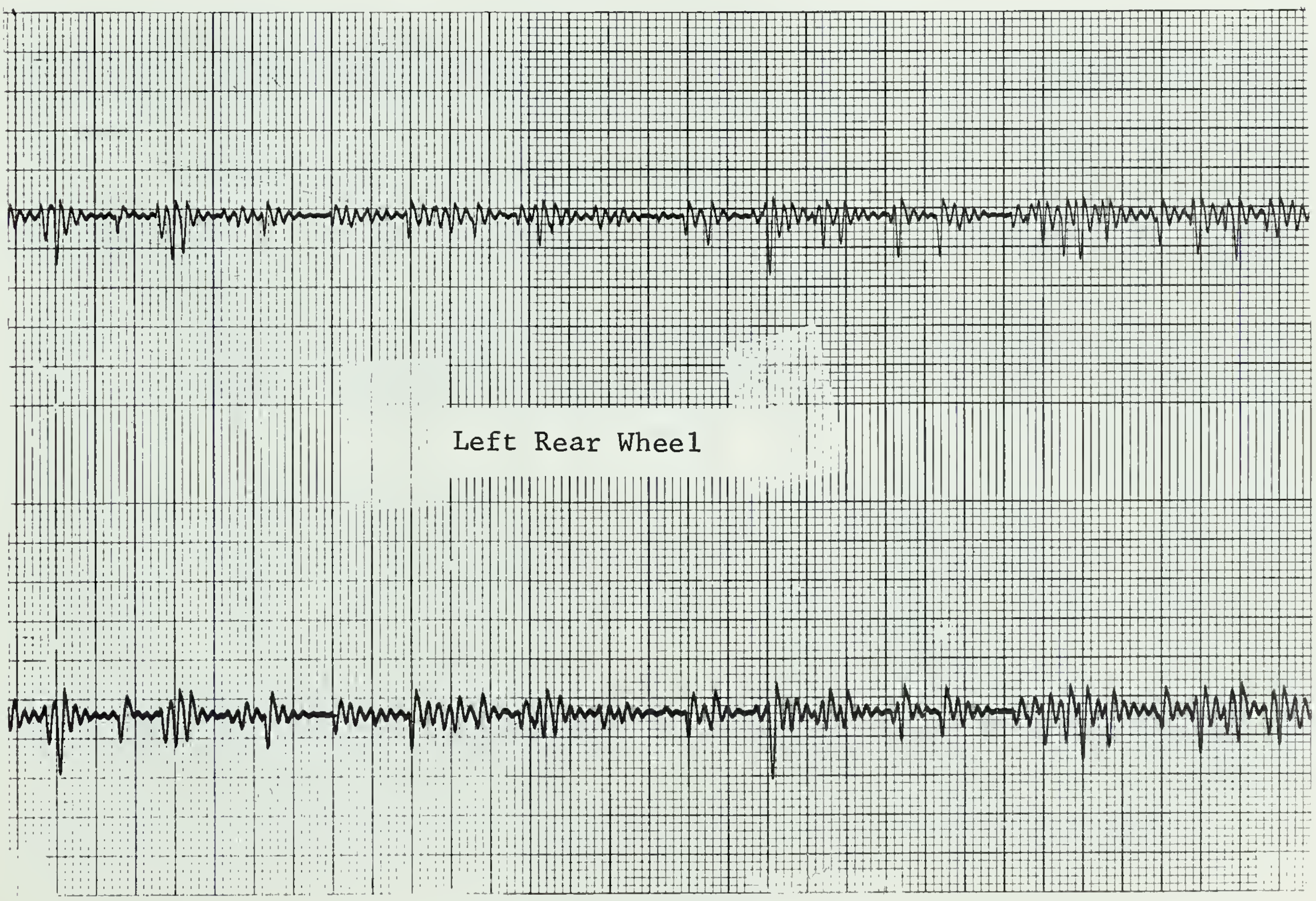

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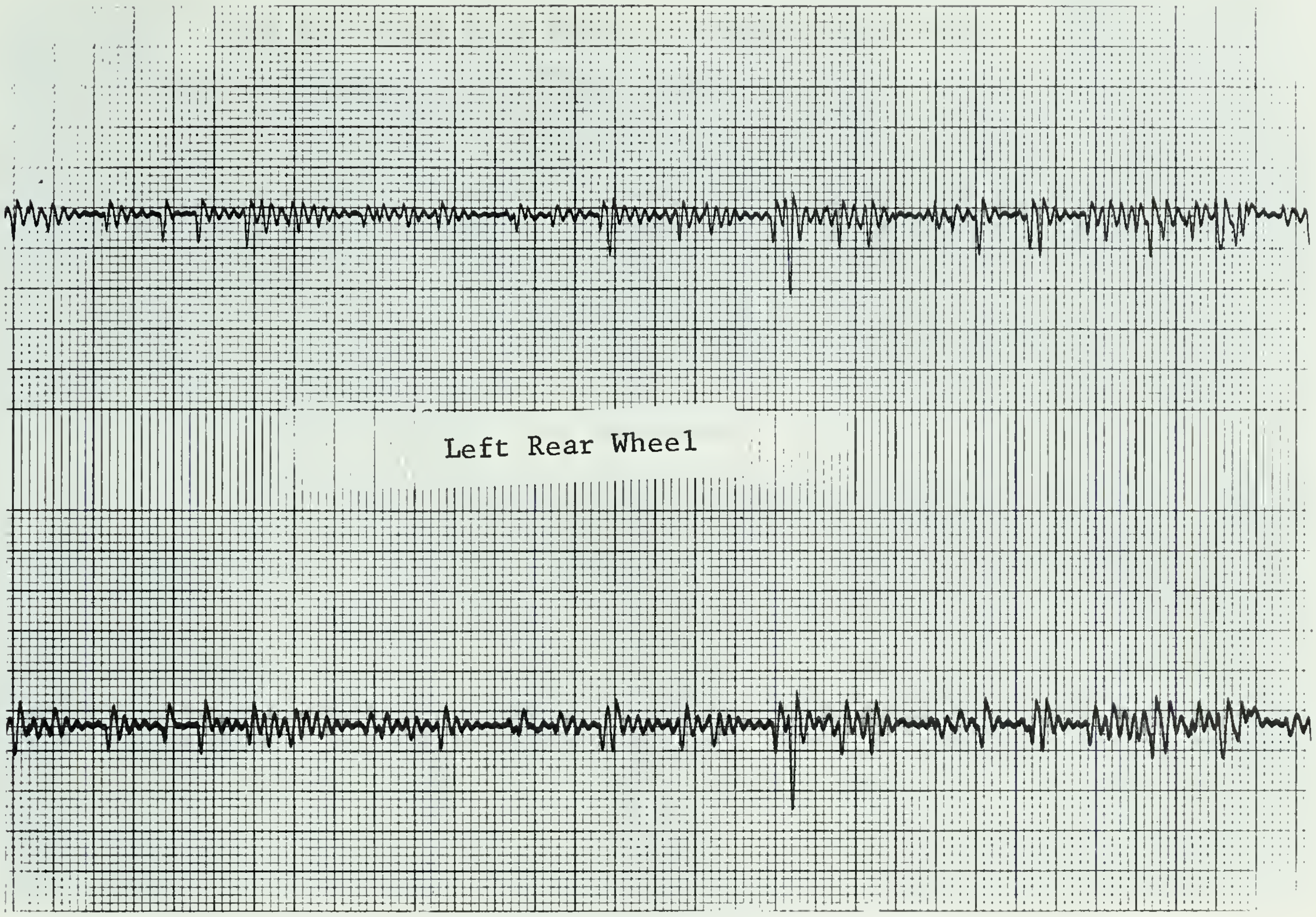
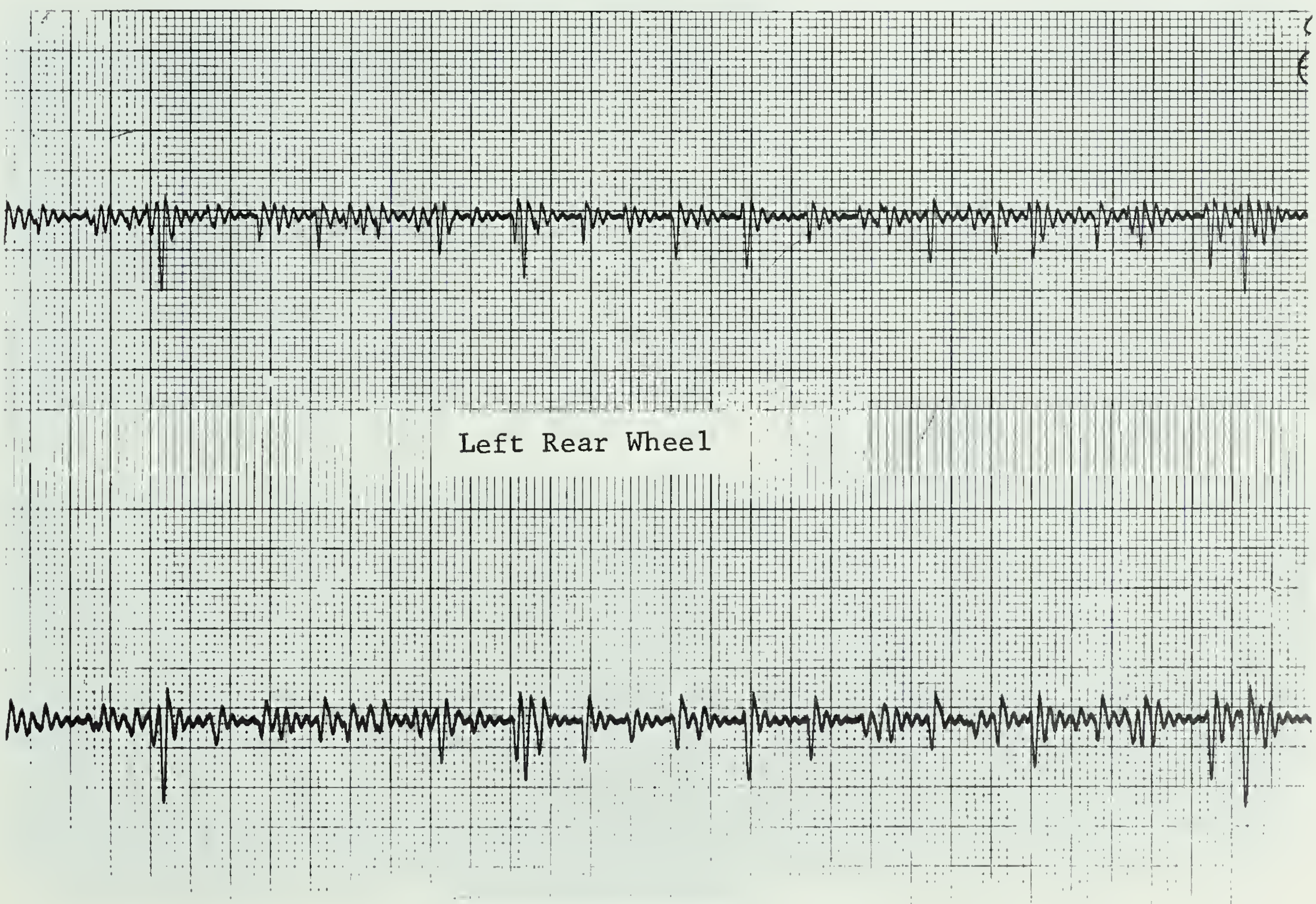
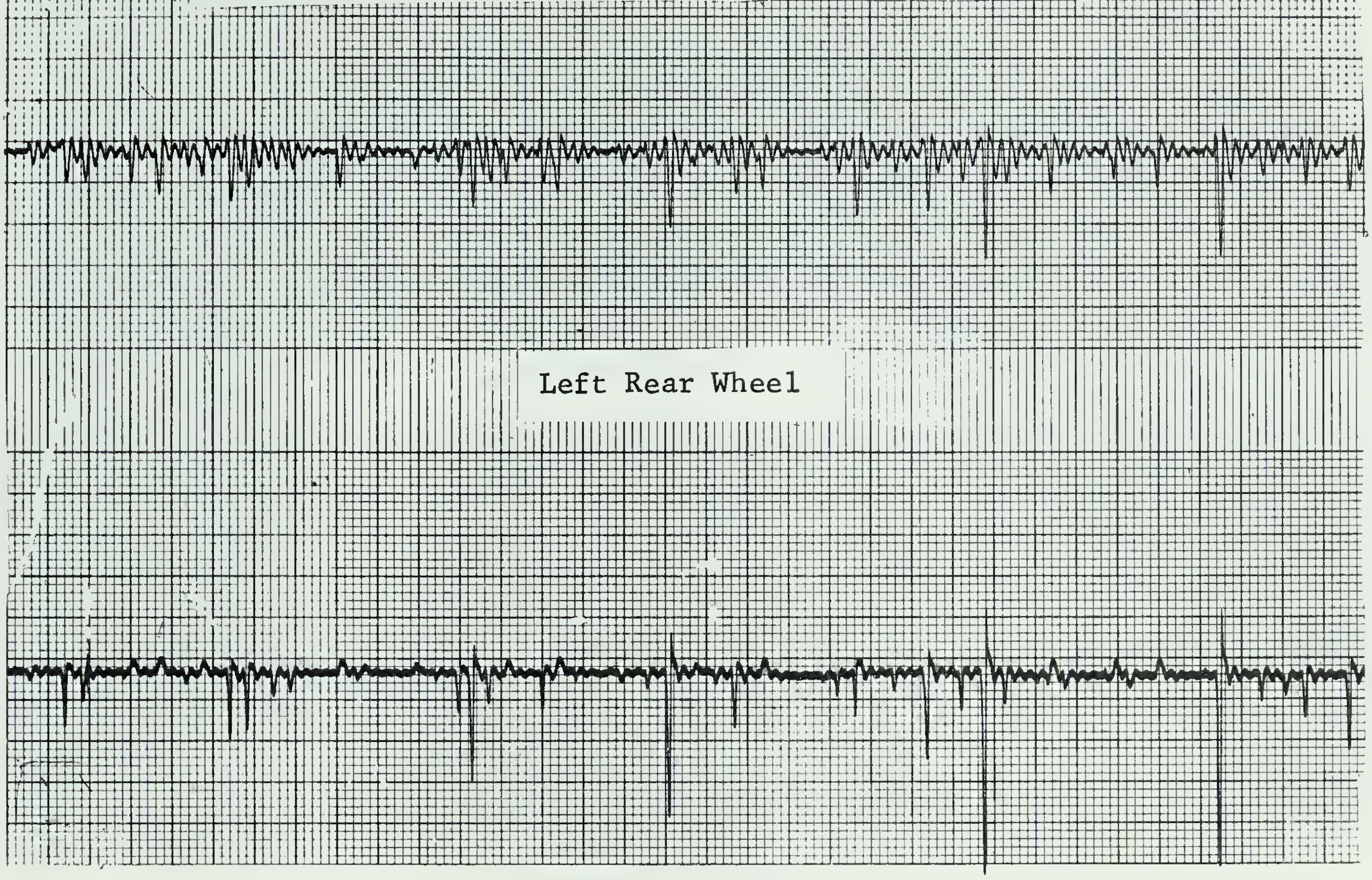


Right Rear Wheel



Right Rear Wheel

Test 4B: Left Rear Wheel and Front Axle Centre.



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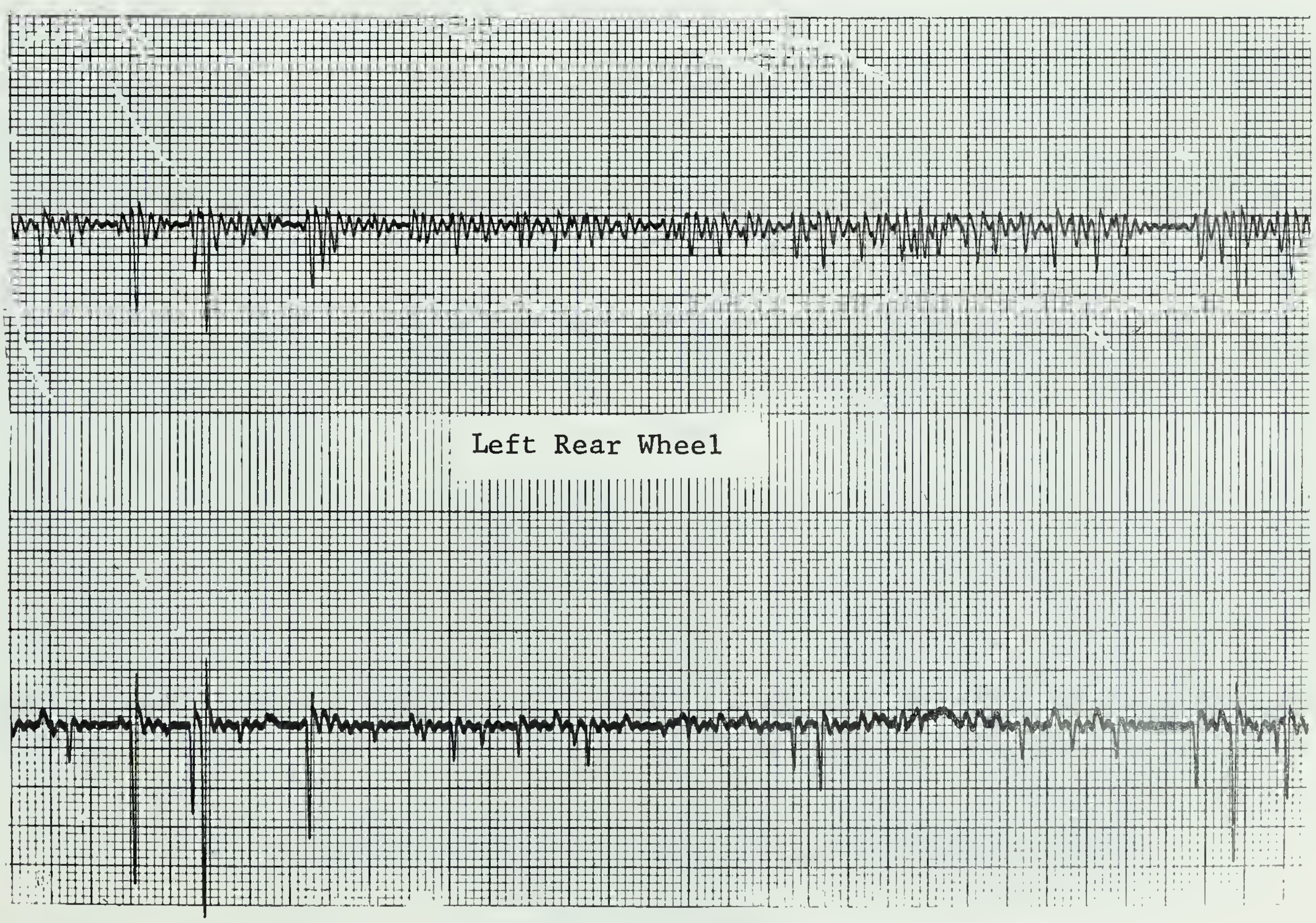

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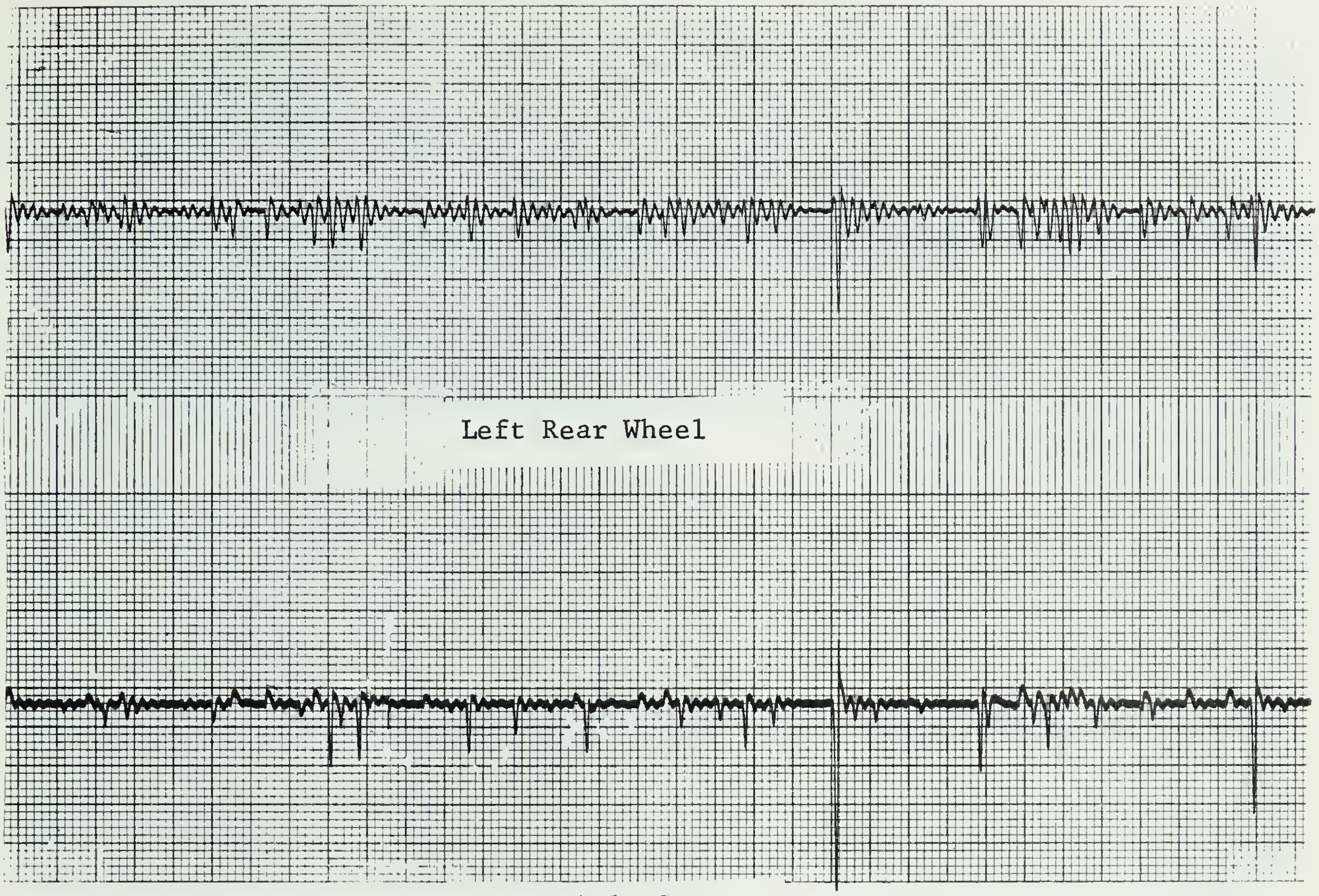
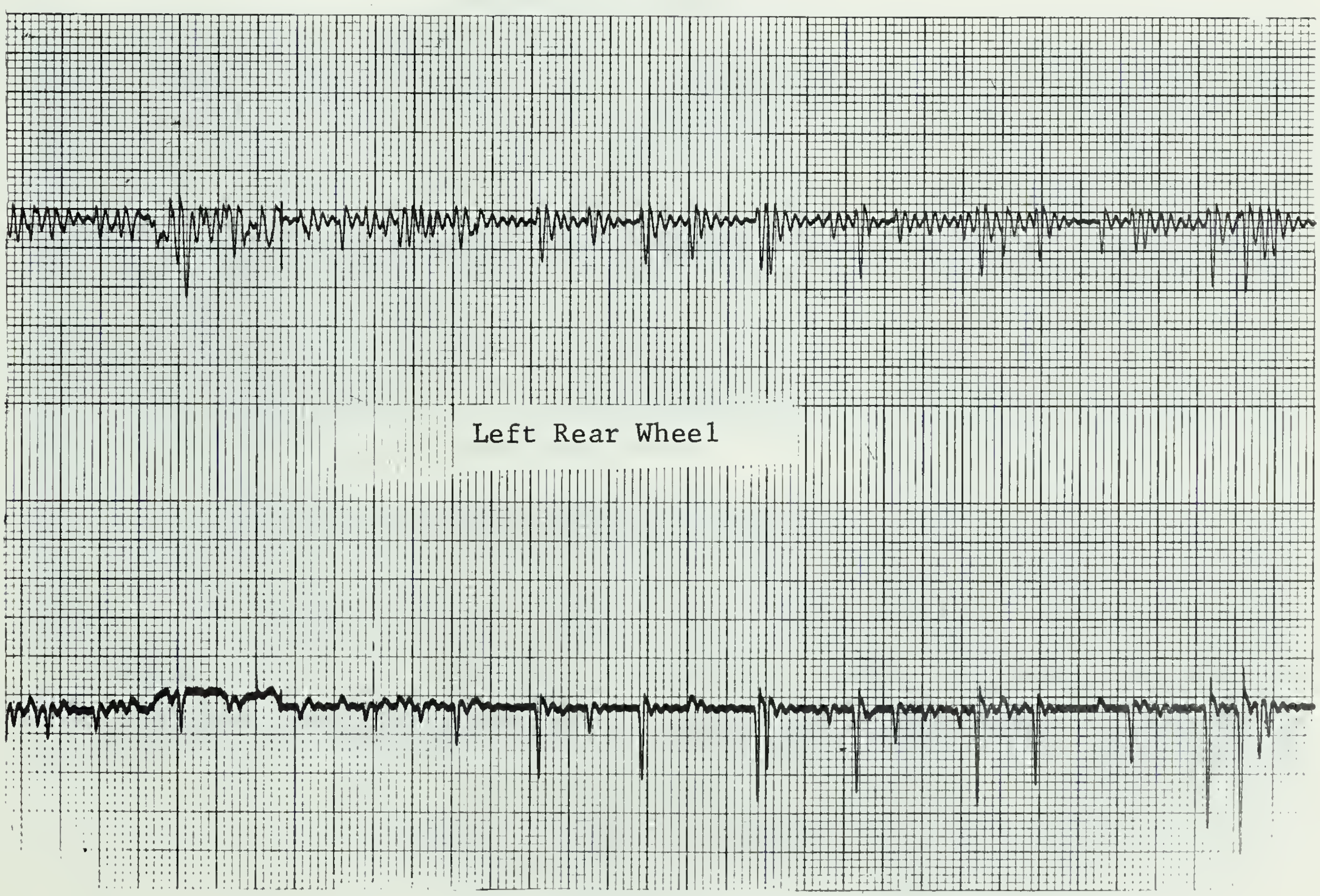
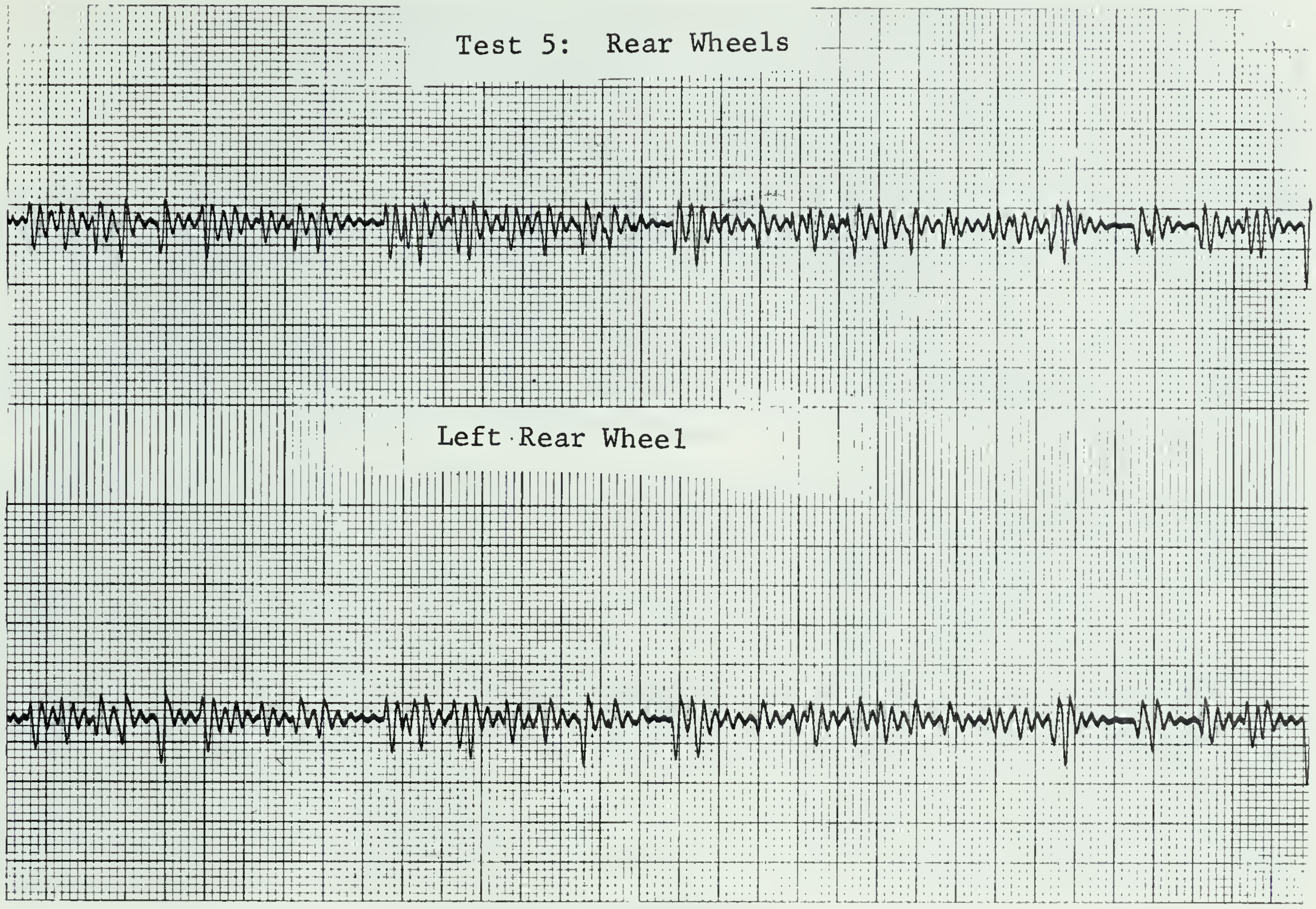


Front Axle Centre

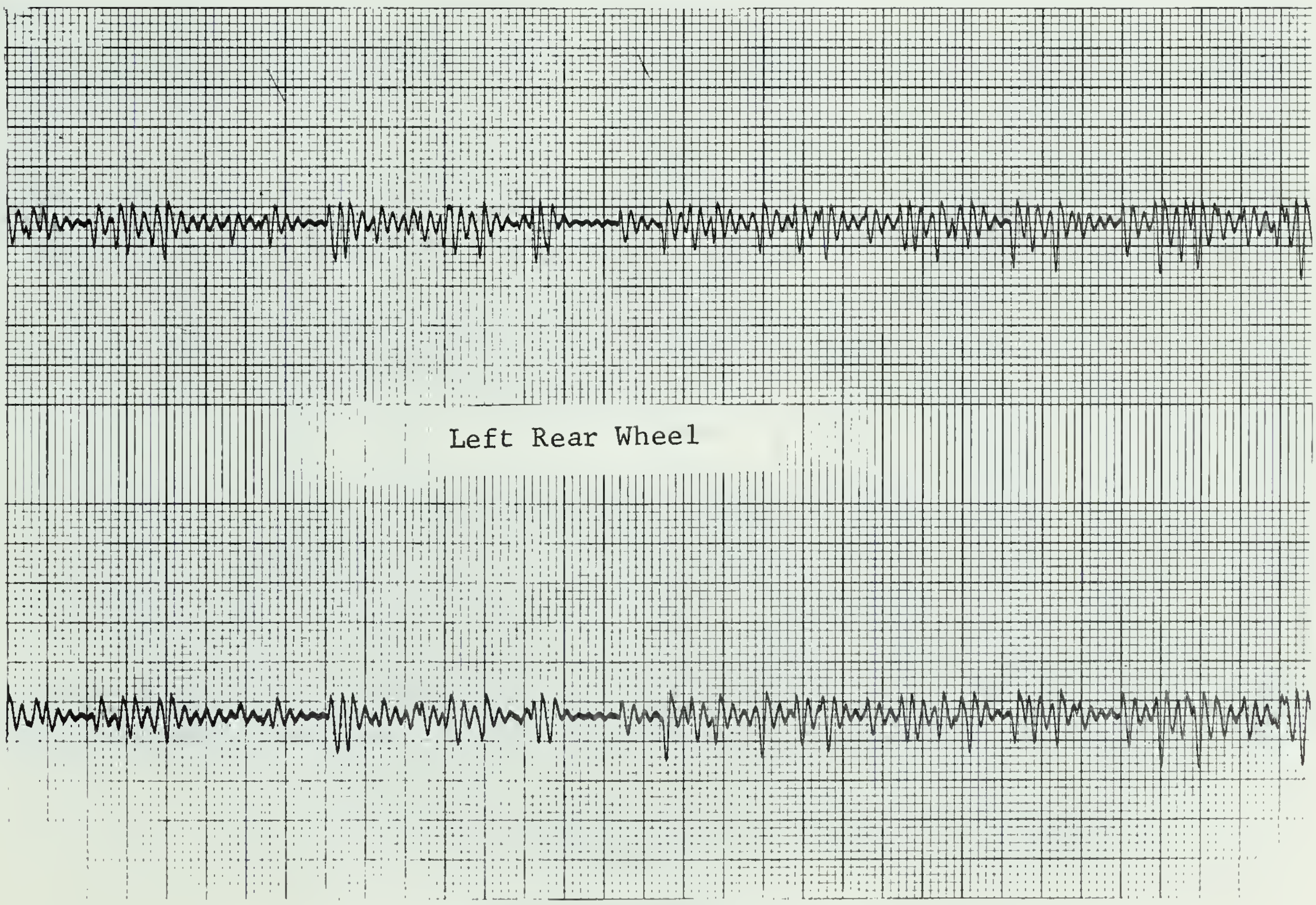


Front Axle Centre

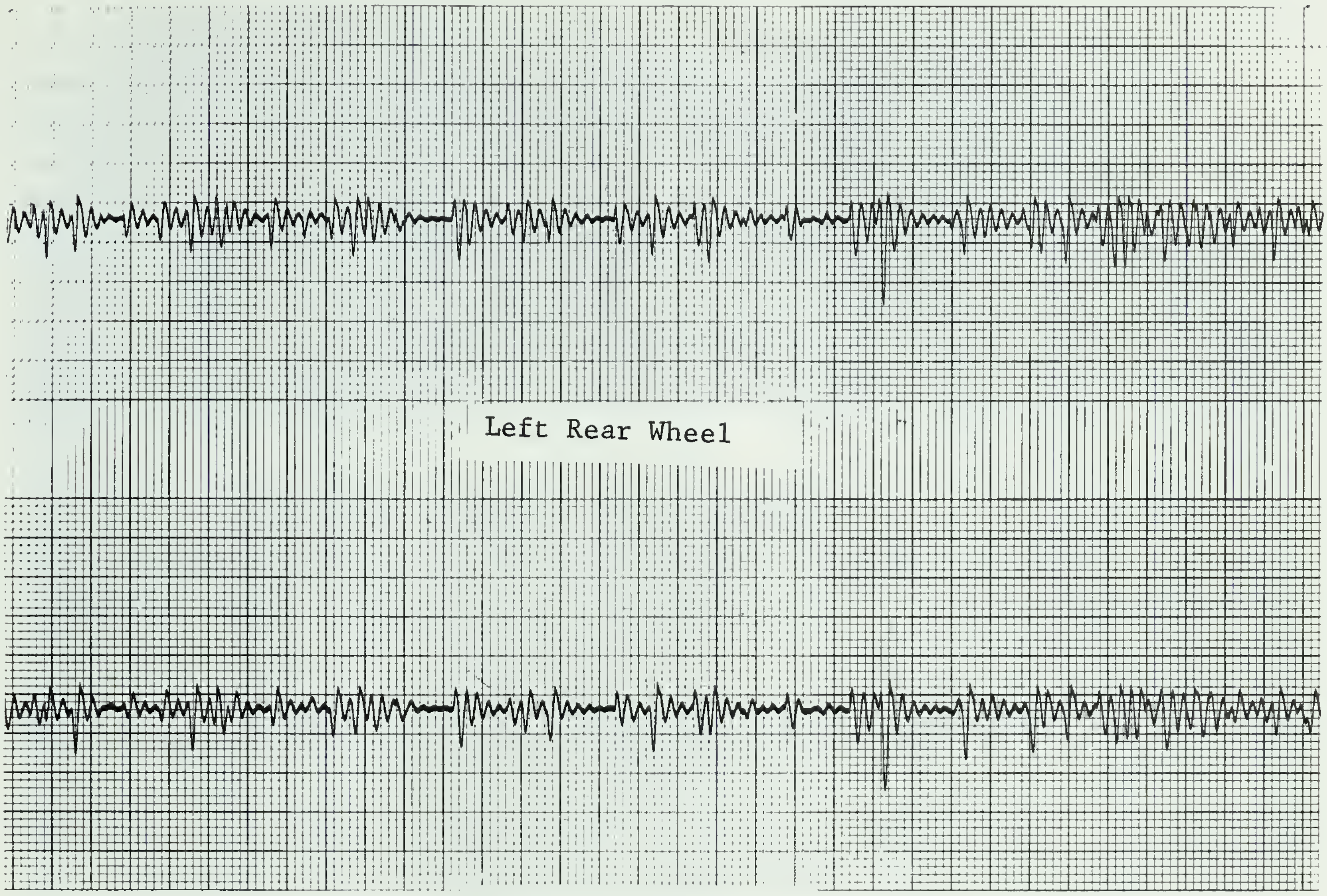
Test 5: Rear Wheels



Right Rear Wheel



Right Rear Wheel



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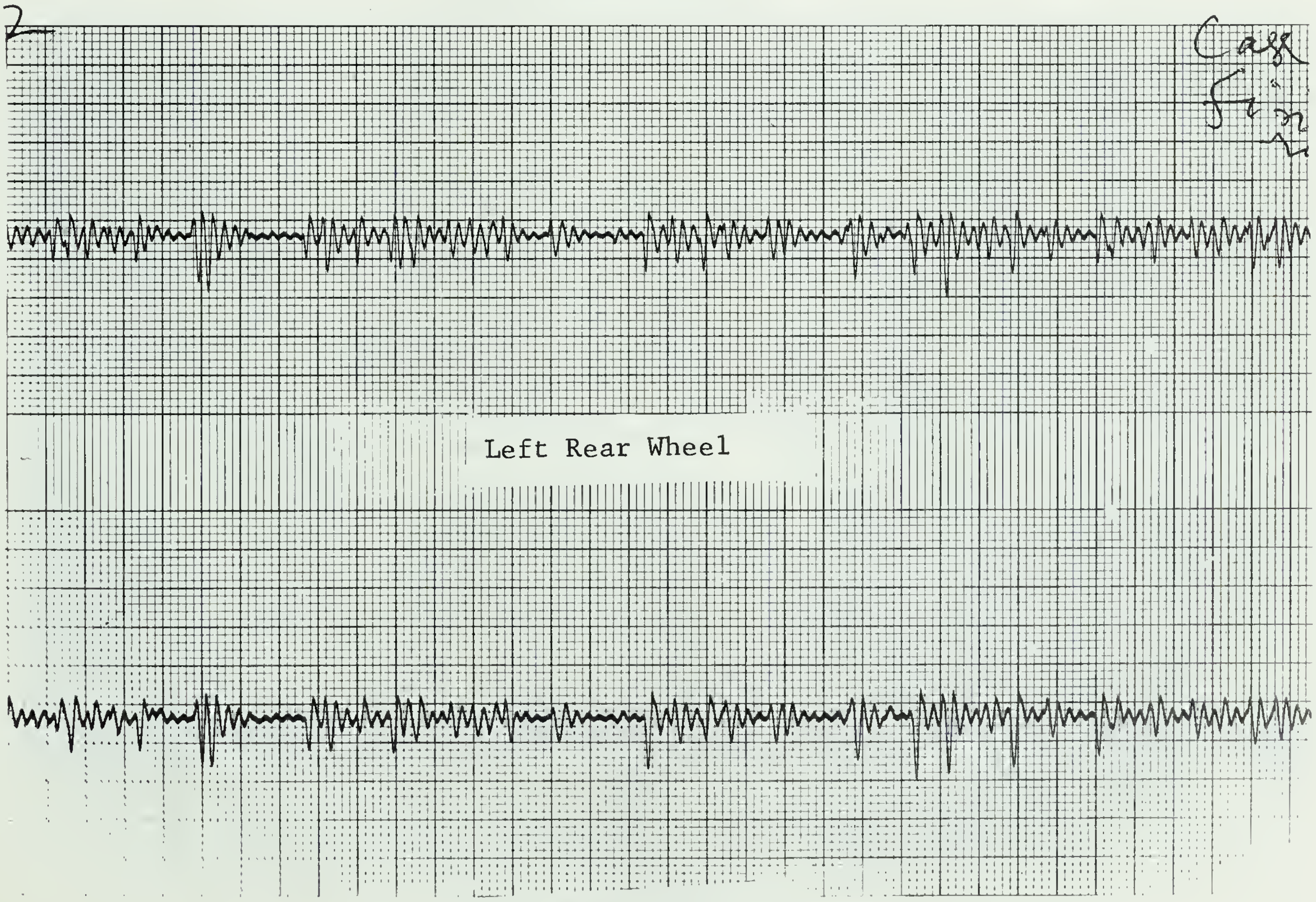

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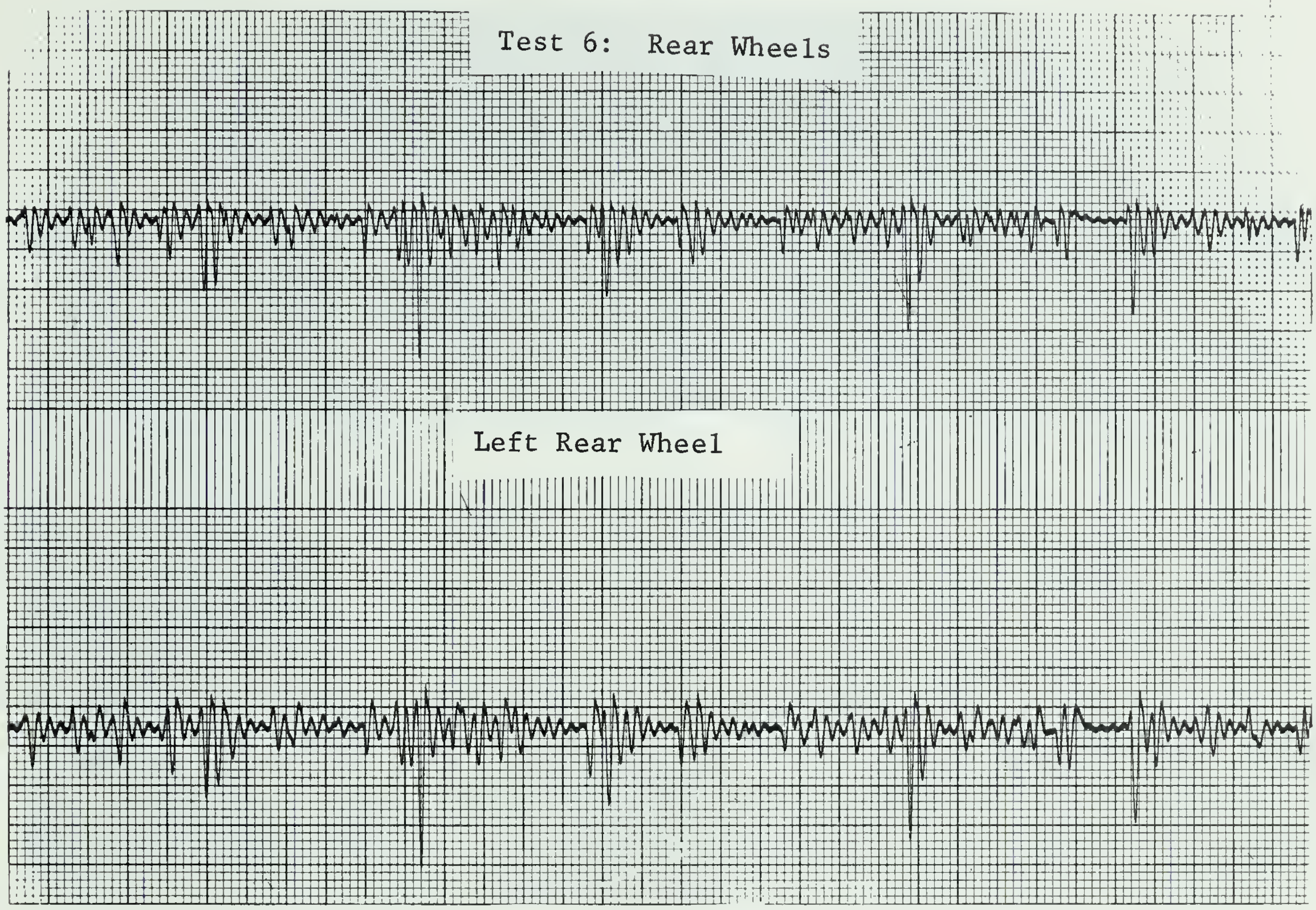

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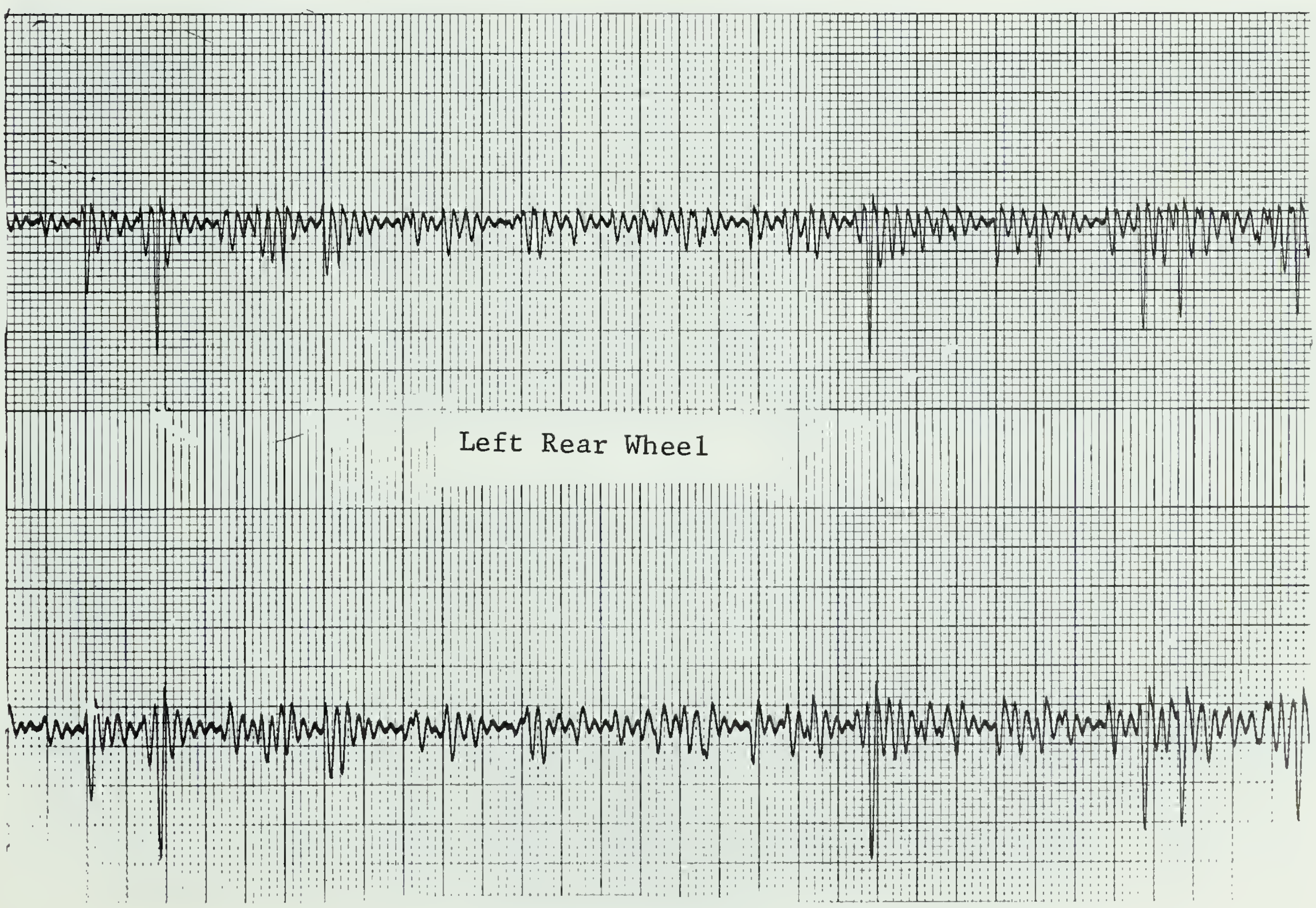

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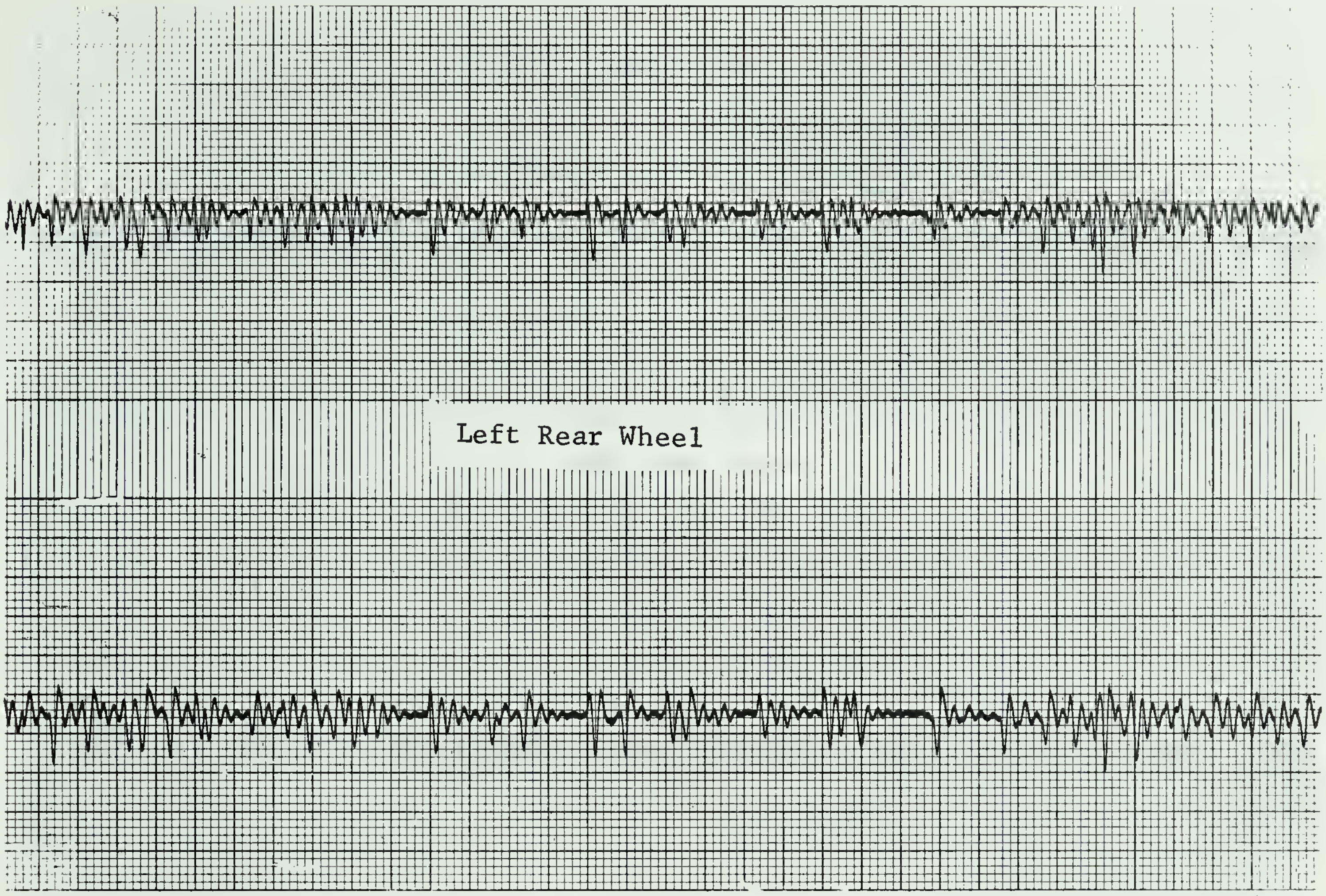

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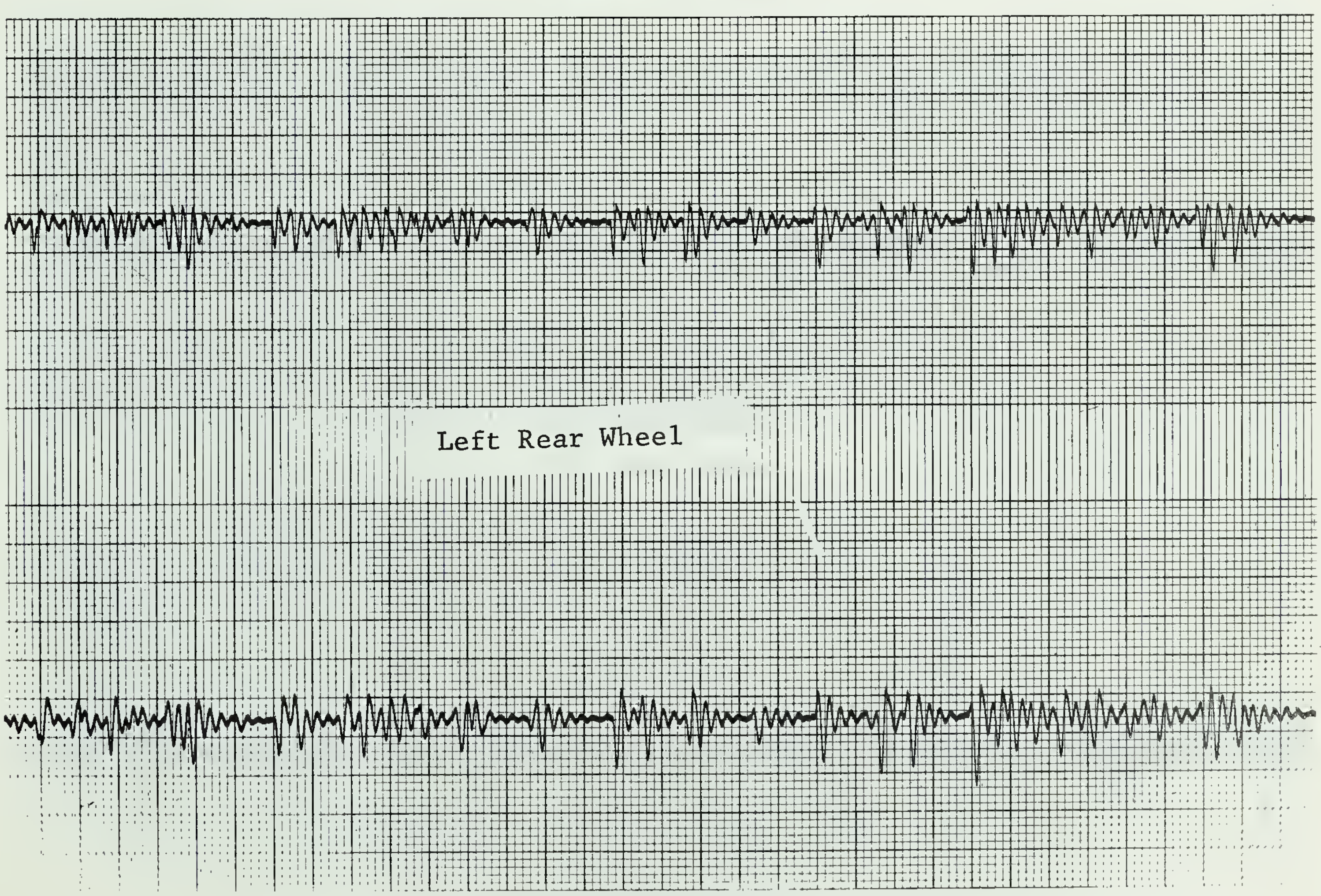

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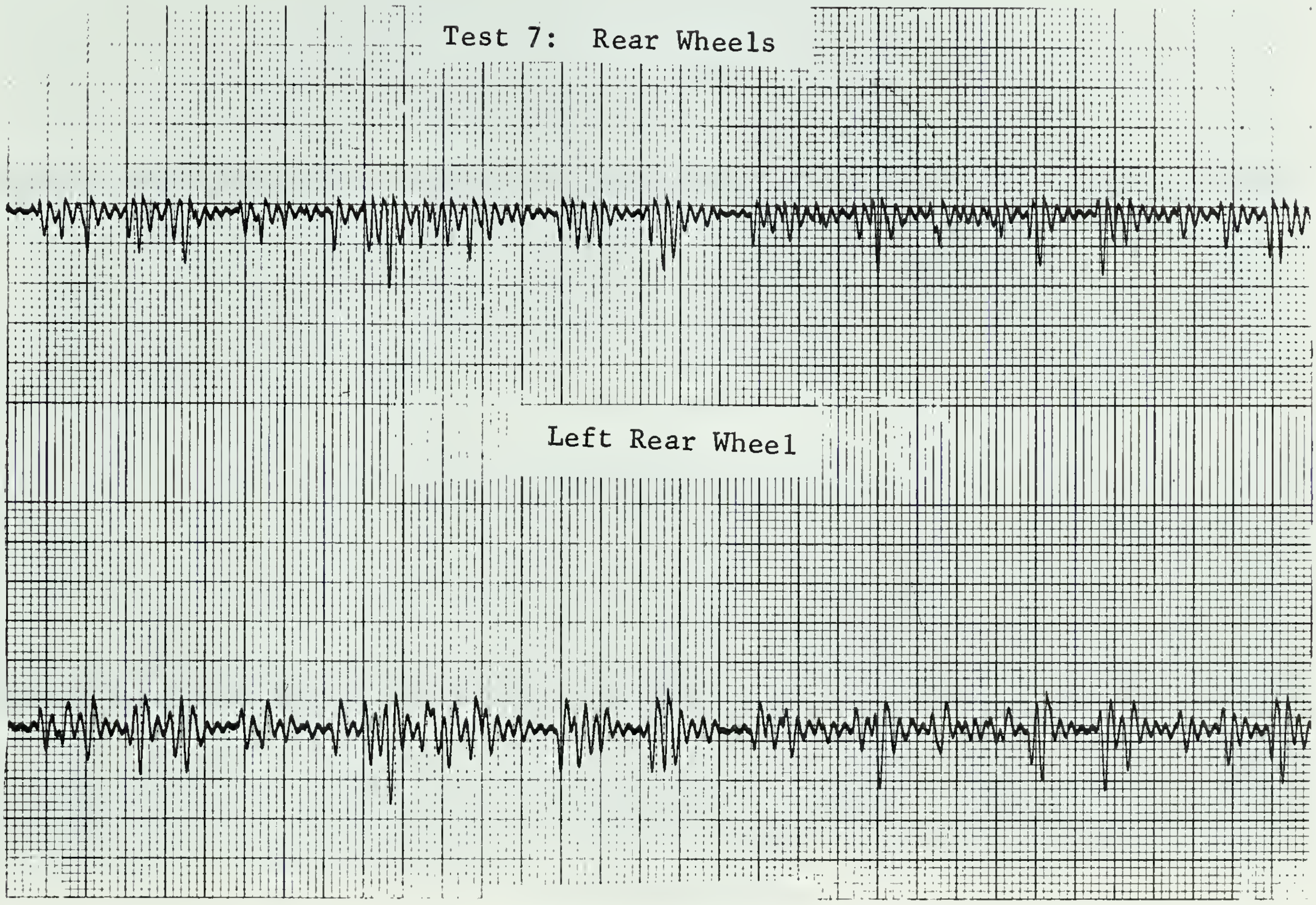

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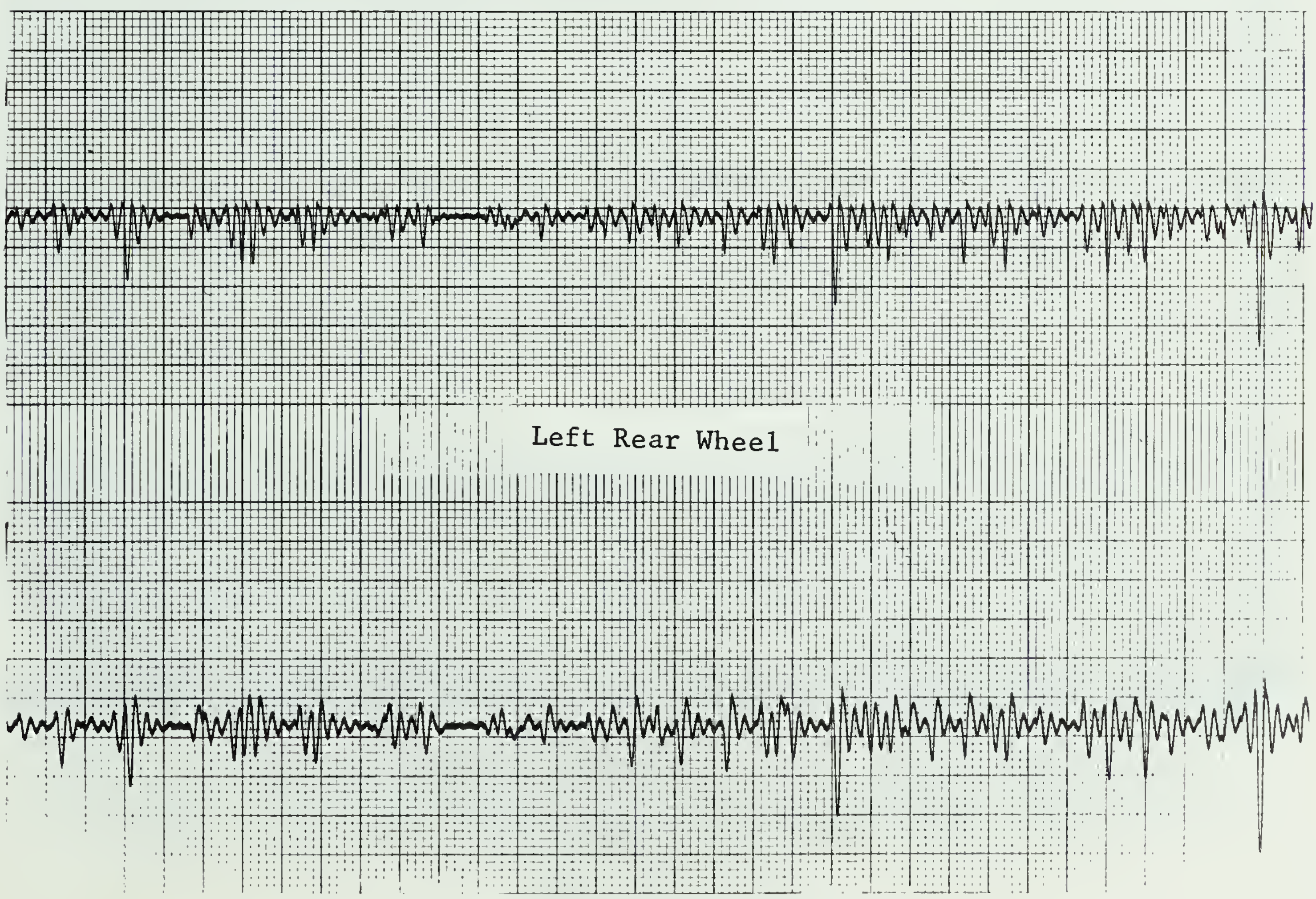

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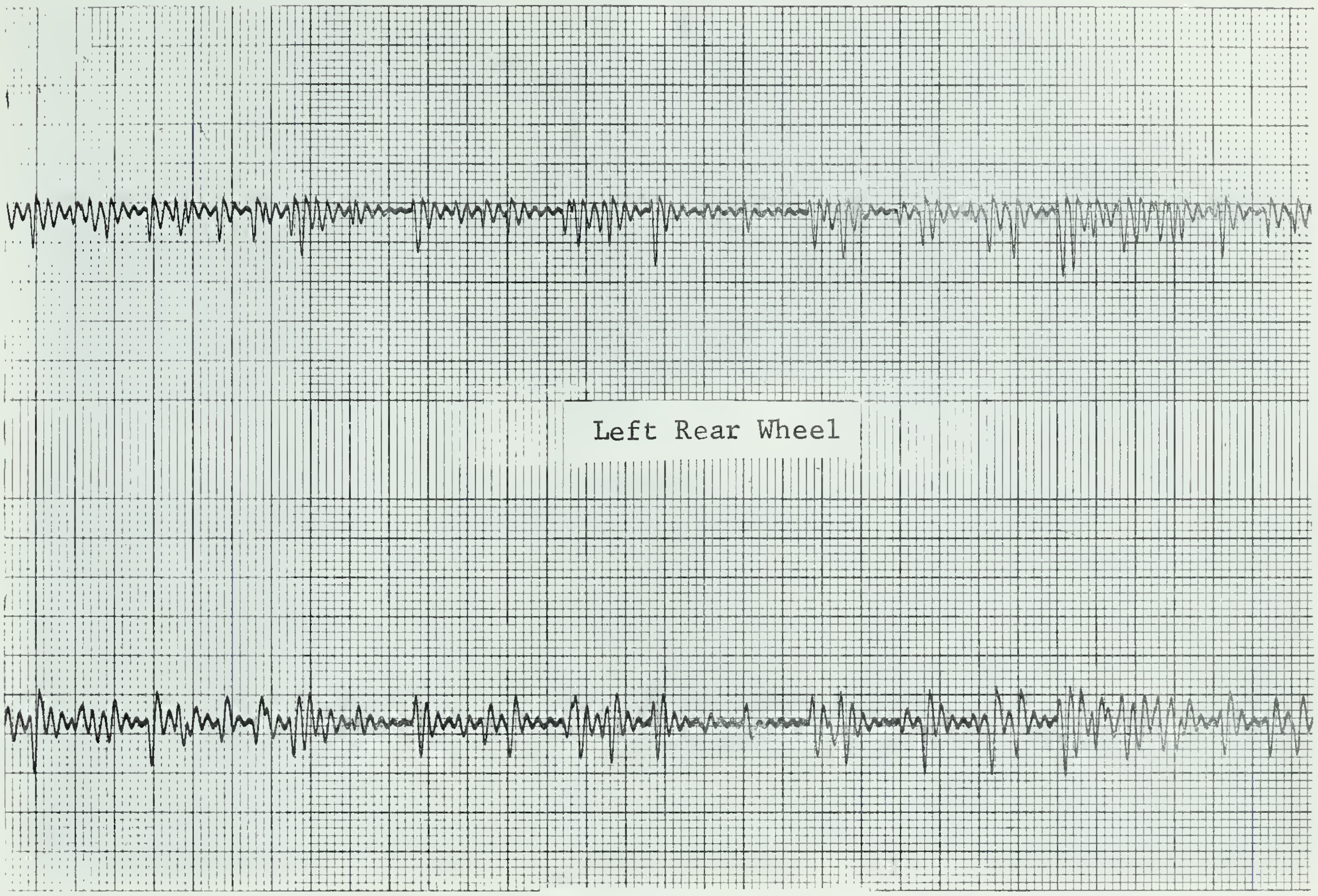

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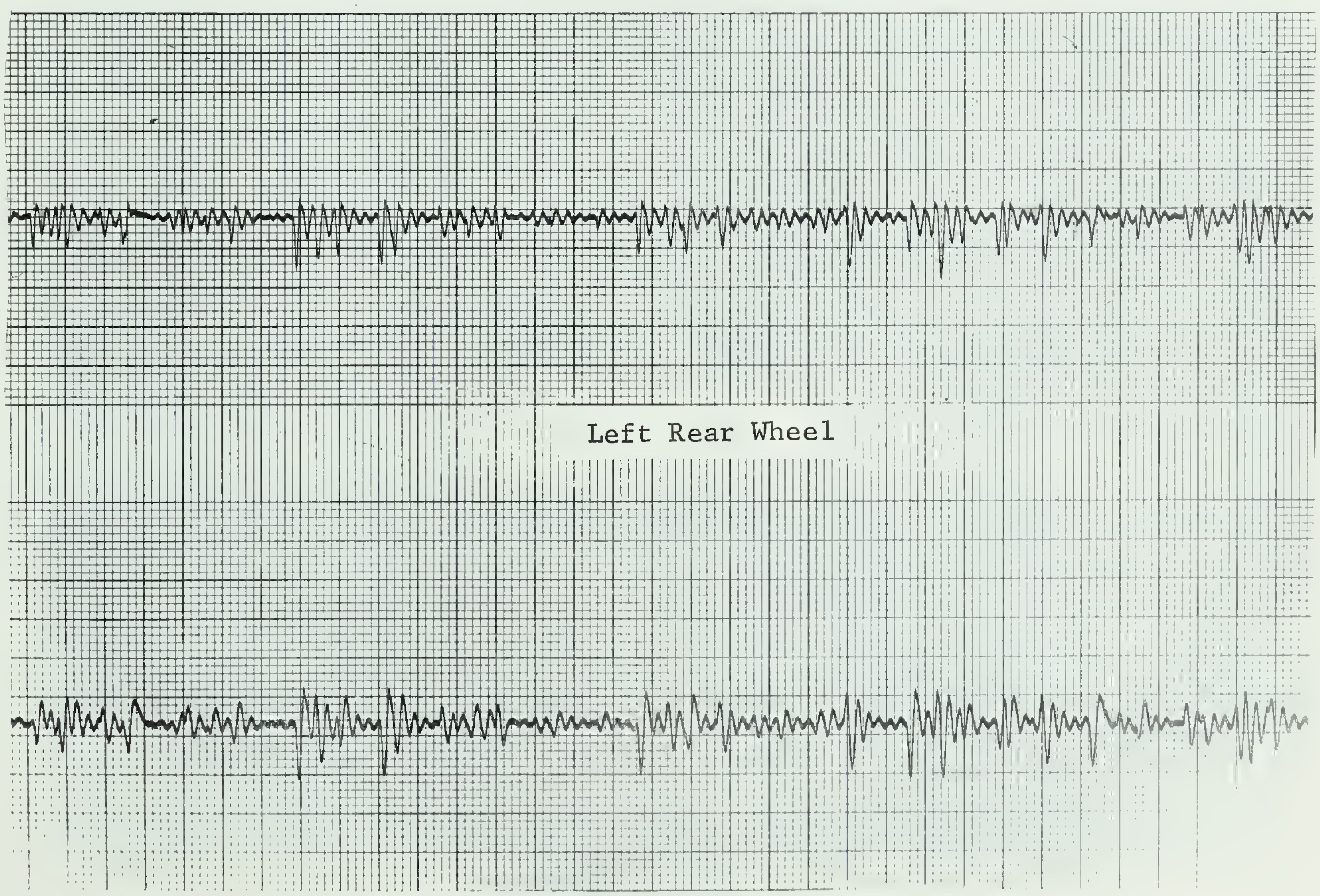
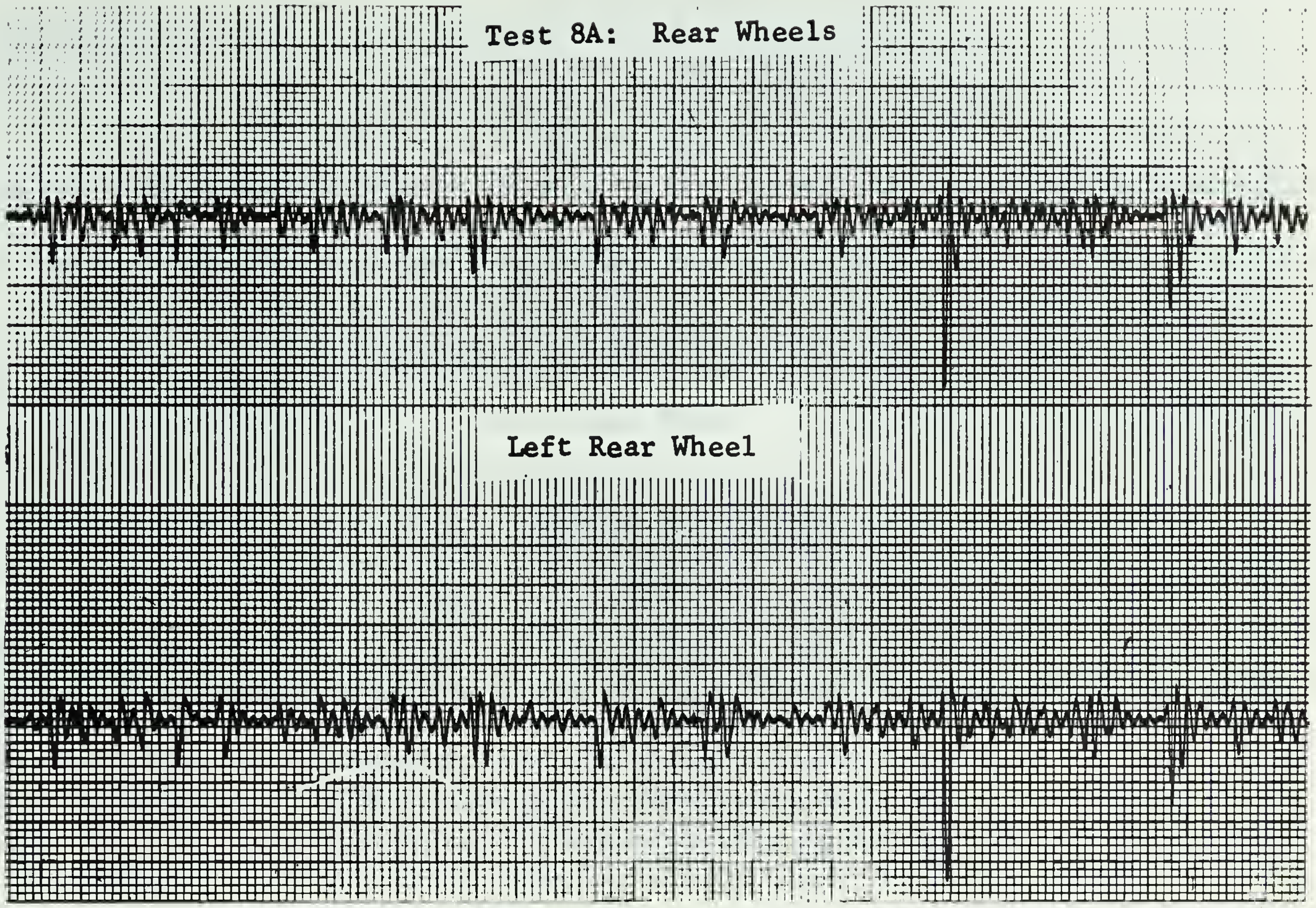
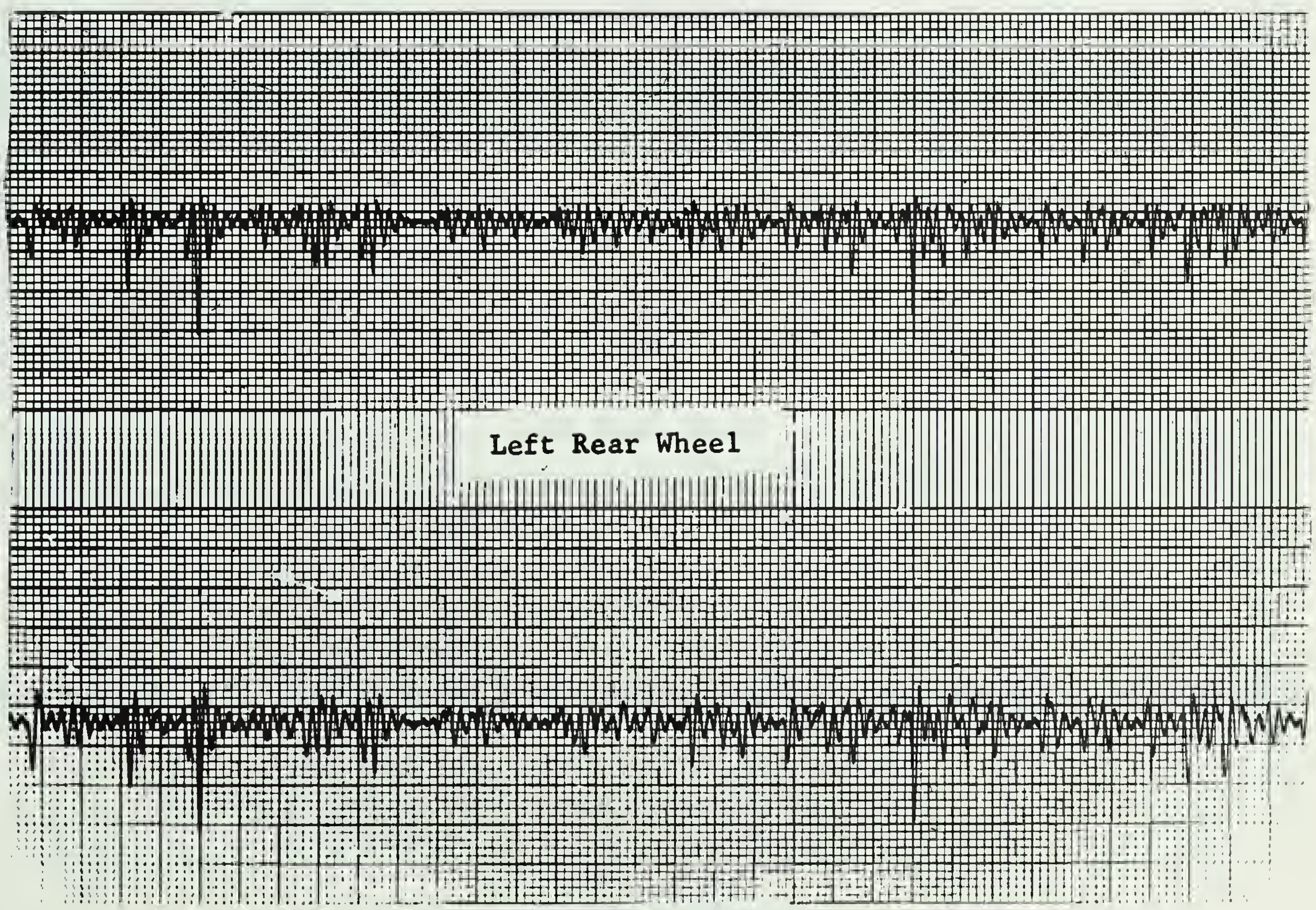


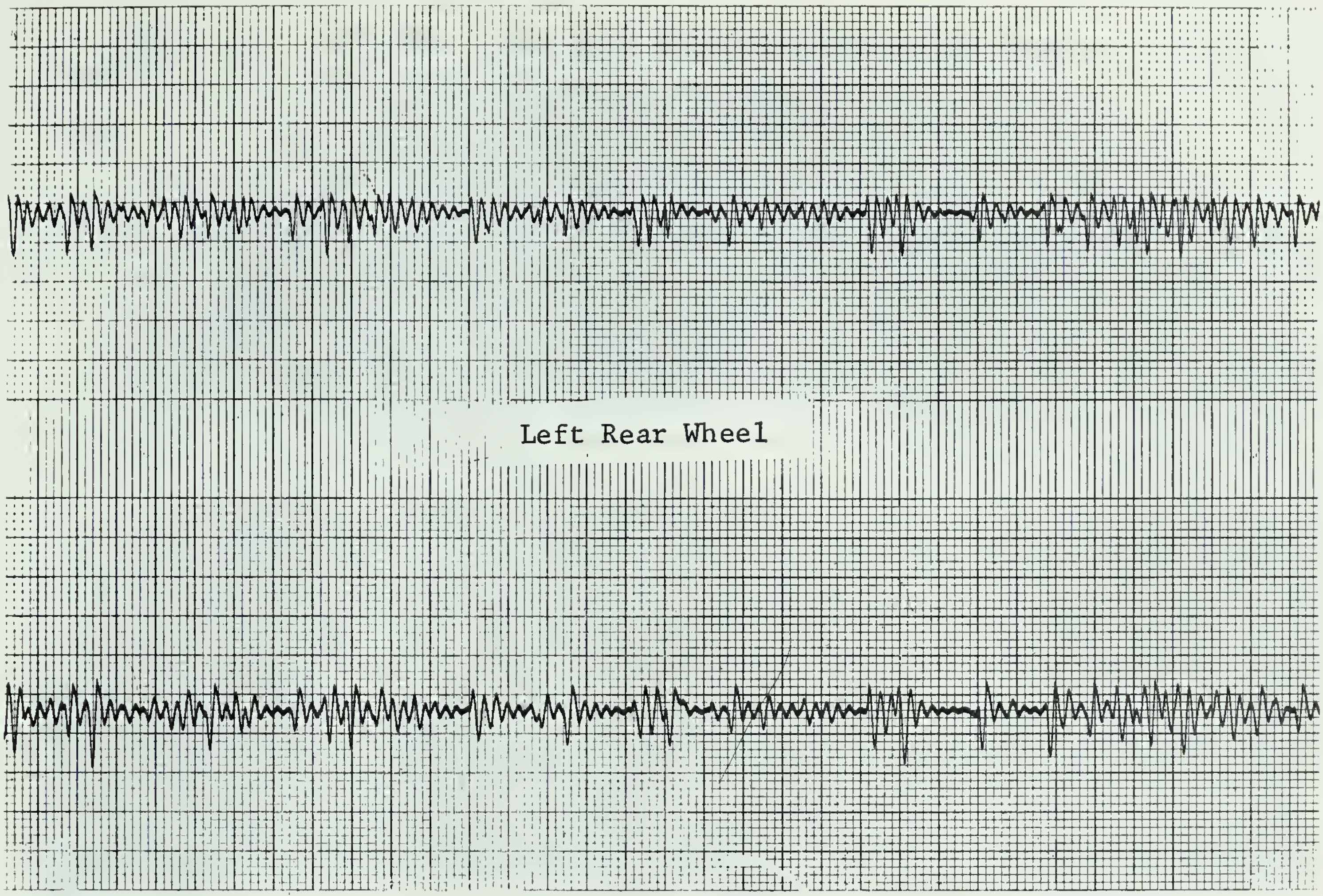
Right Rear Wheel



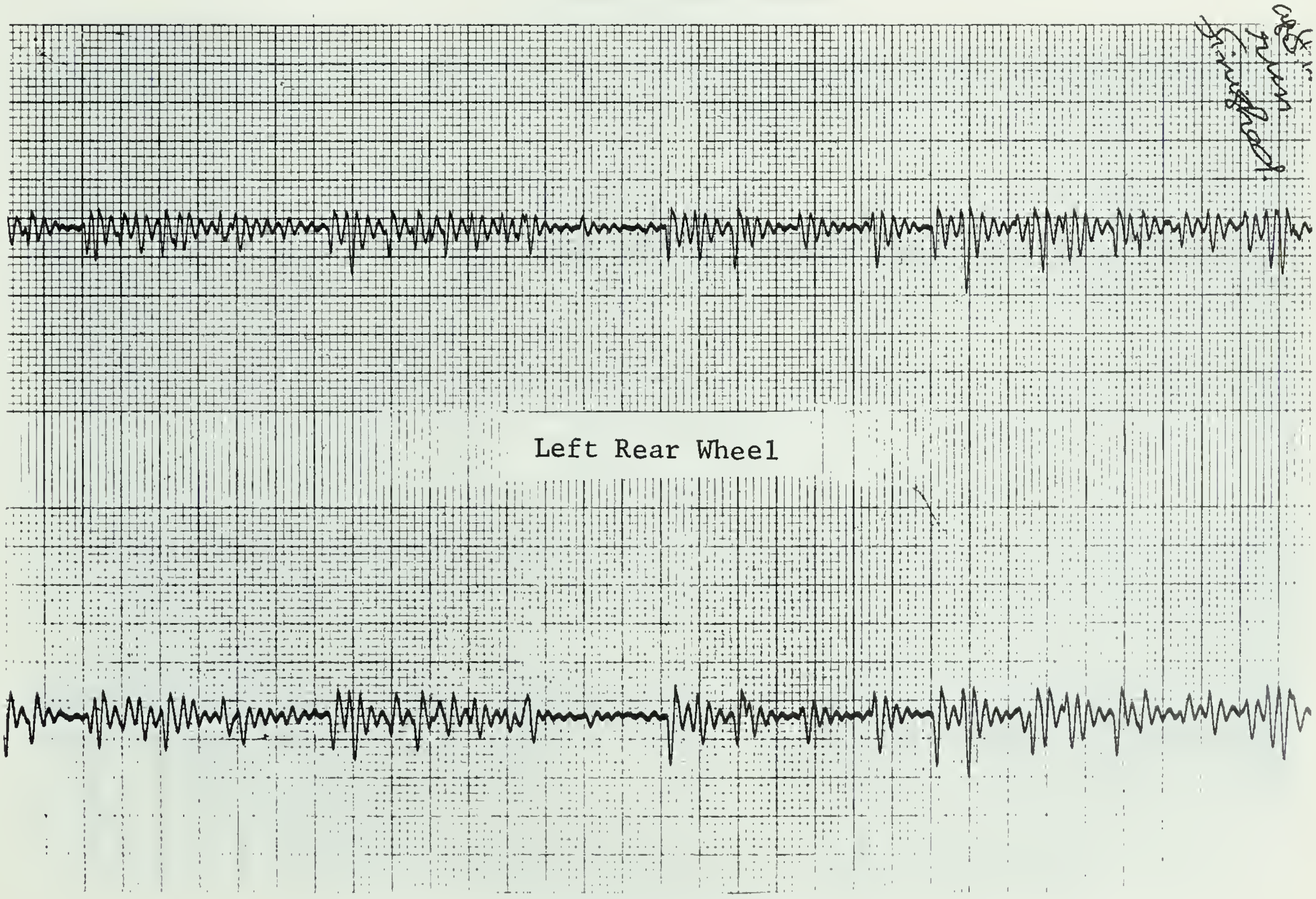
Right Rear Wheel



Right Rear Wheel

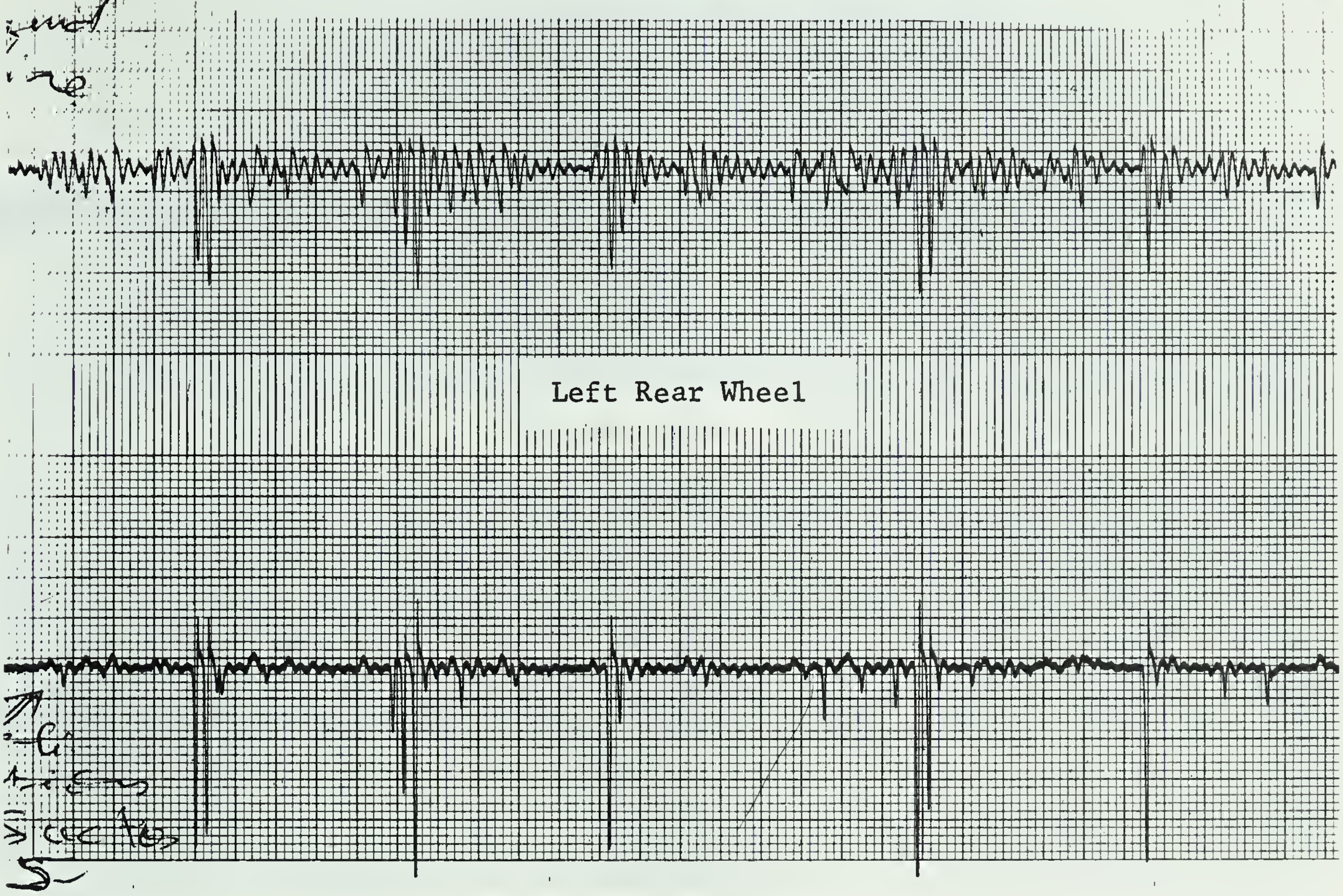


Right Rear Wheel

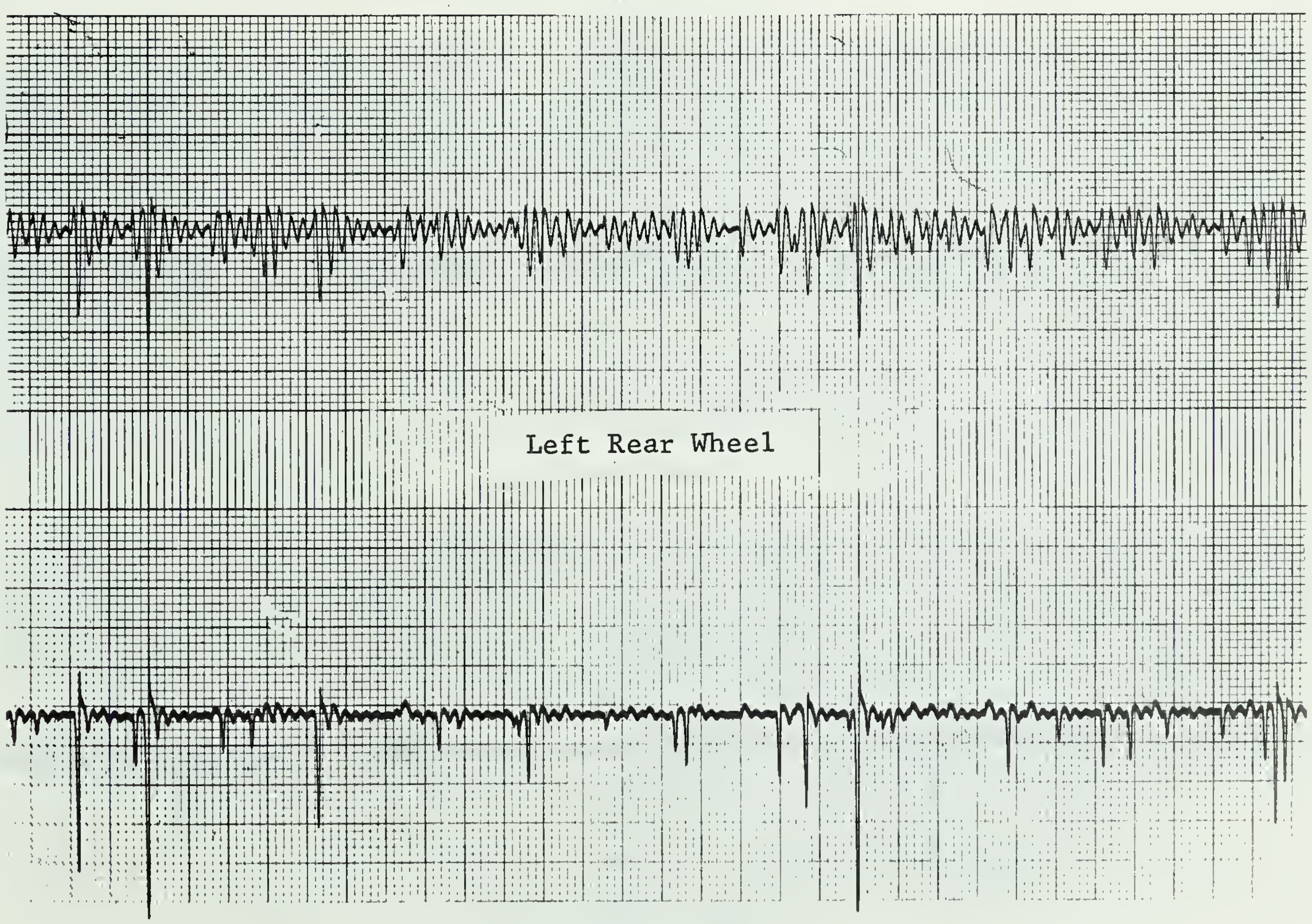


Right Rear Wheel

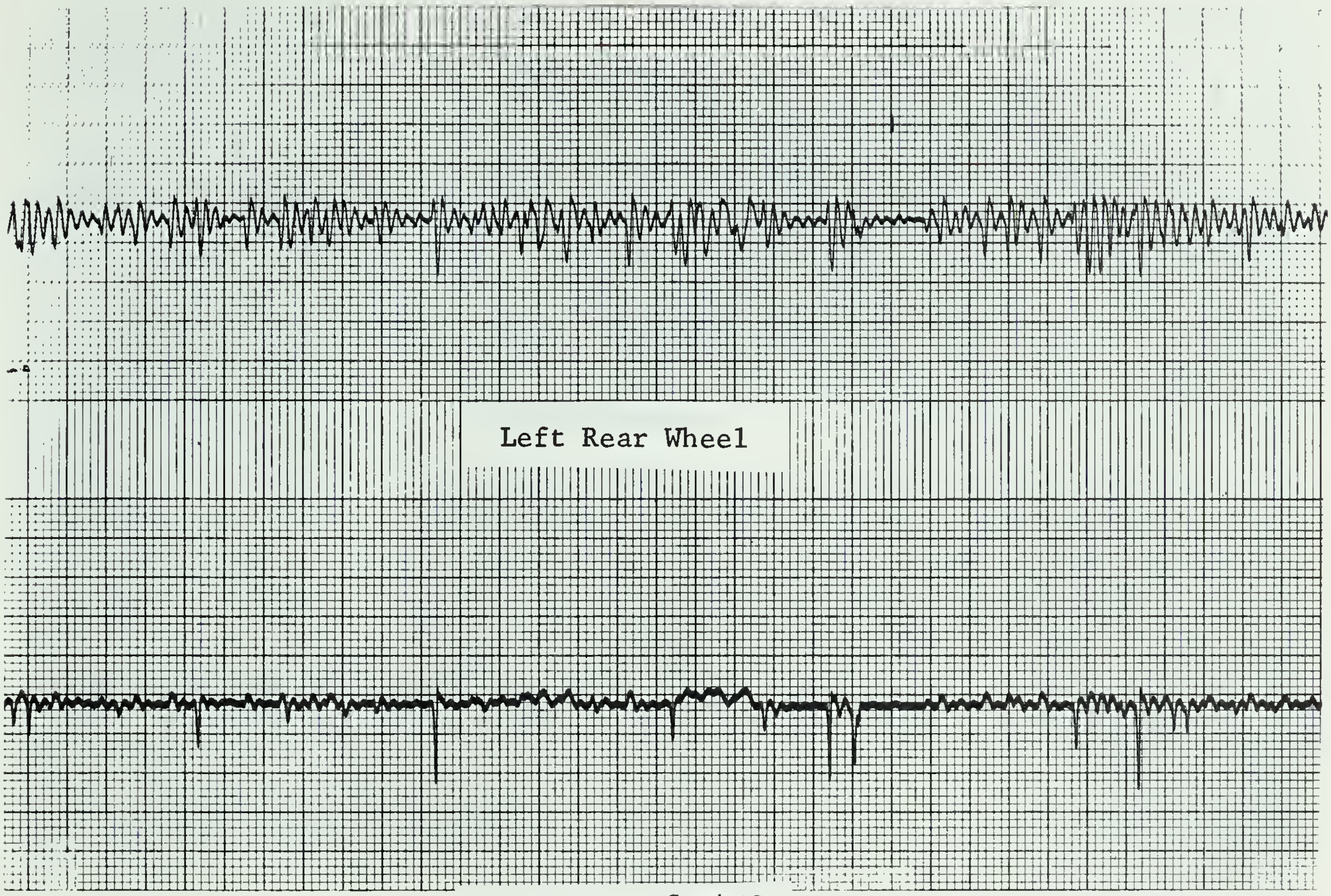
Test 8B: Left Rear Wheel and Front Axle Centre.



Front Axle Centre

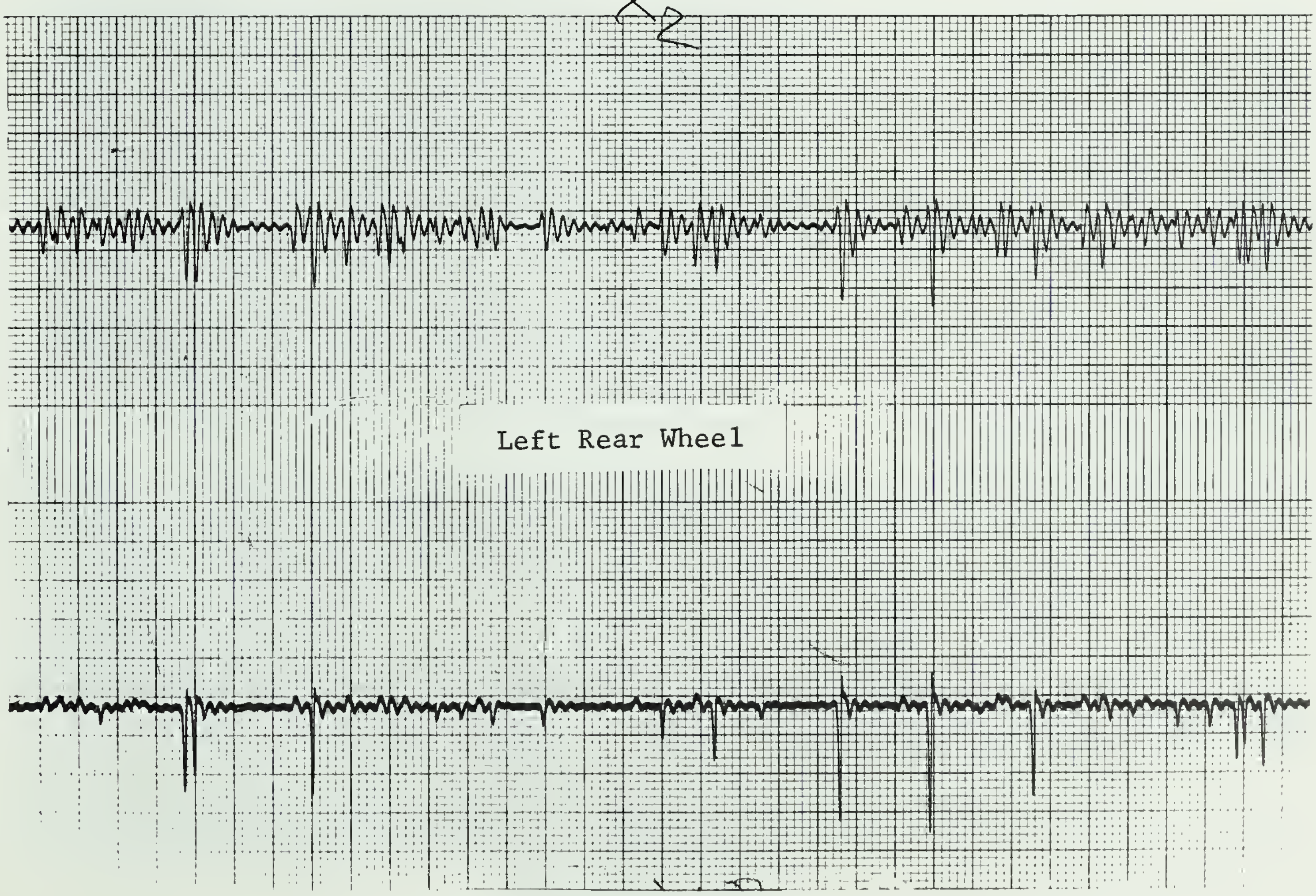


Front Axle Centre



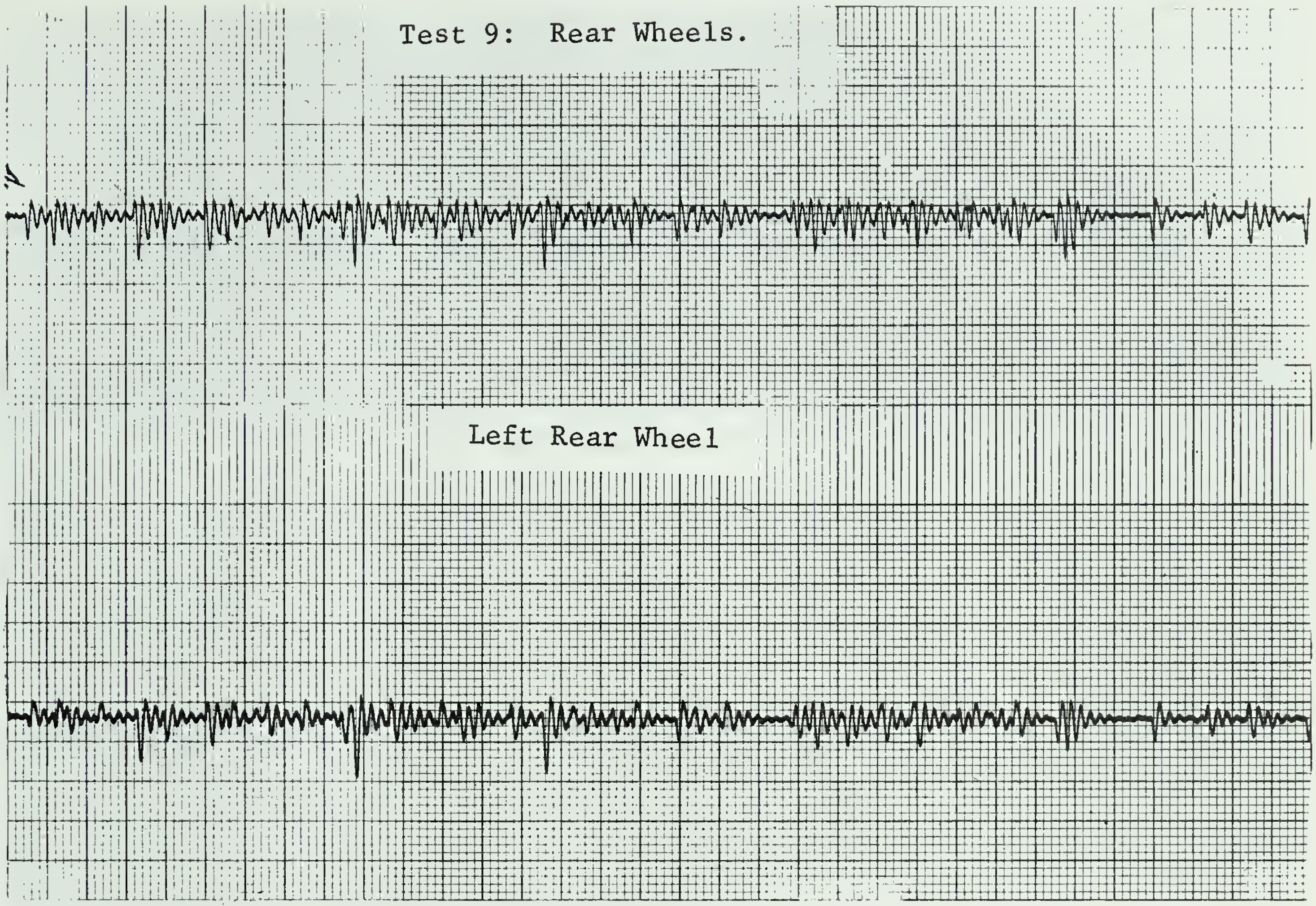
Front Axle Centre

X2

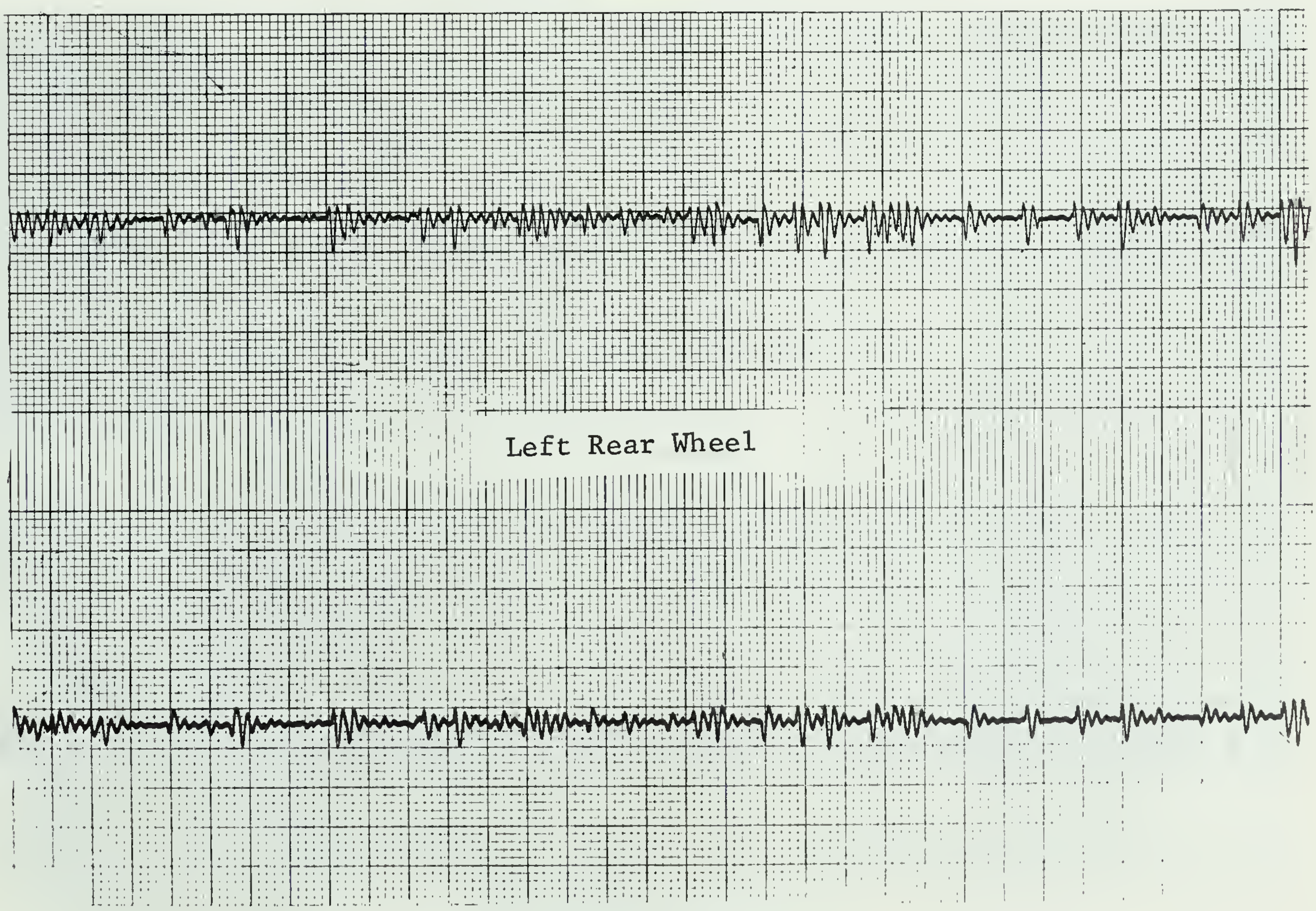


Front Axle Centre

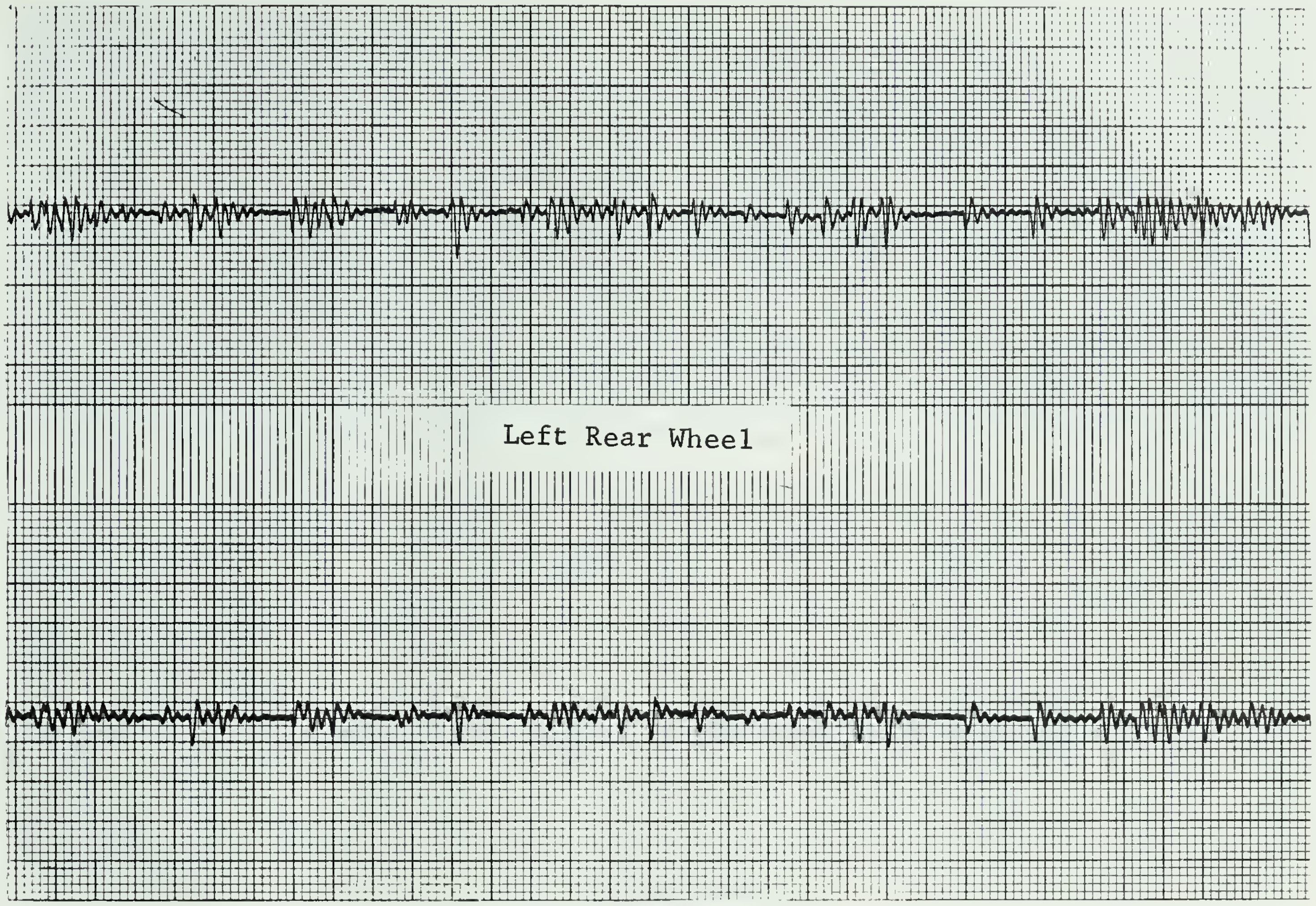
Test 9: Rear Wheels.



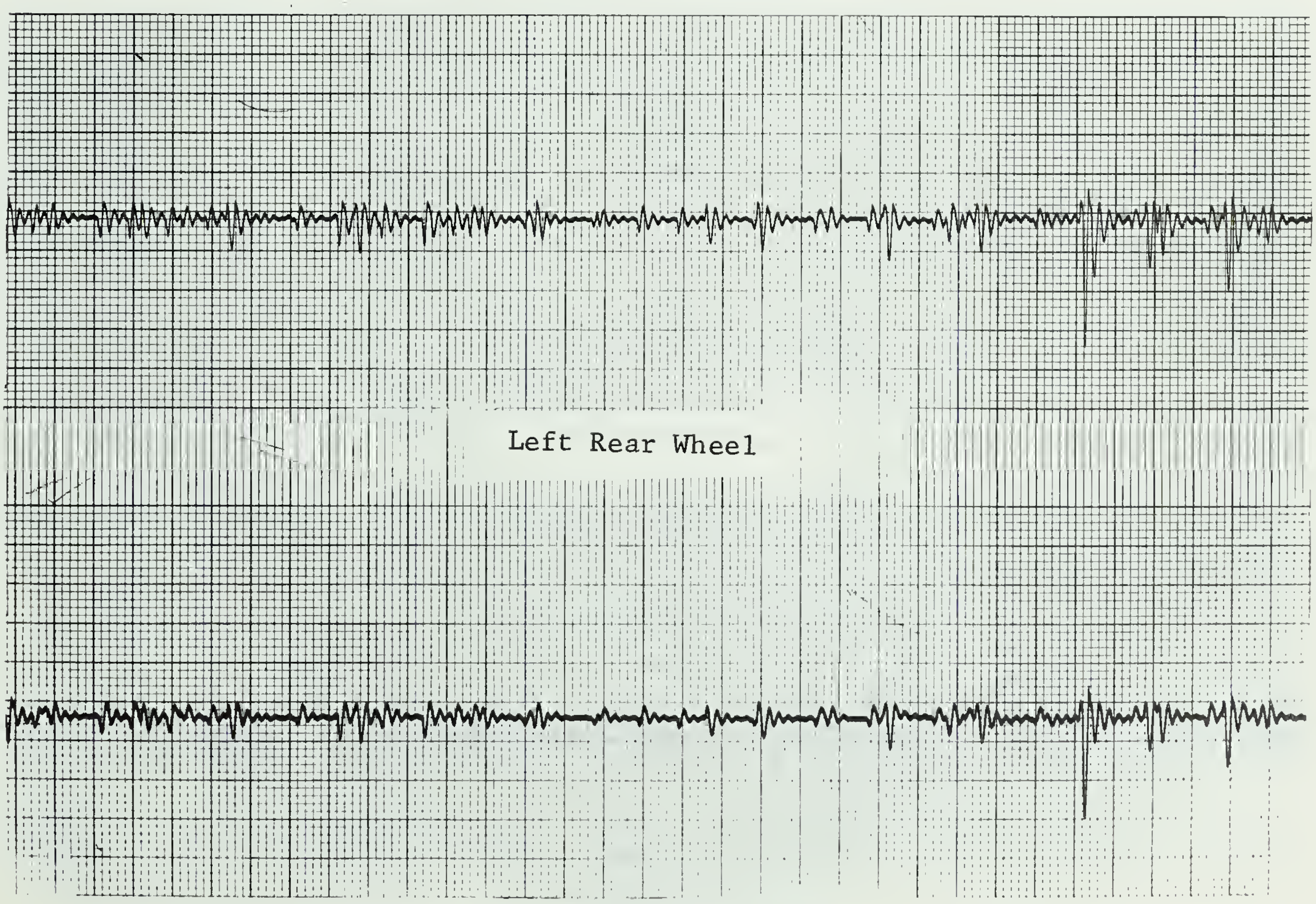
Right Rear Wheel



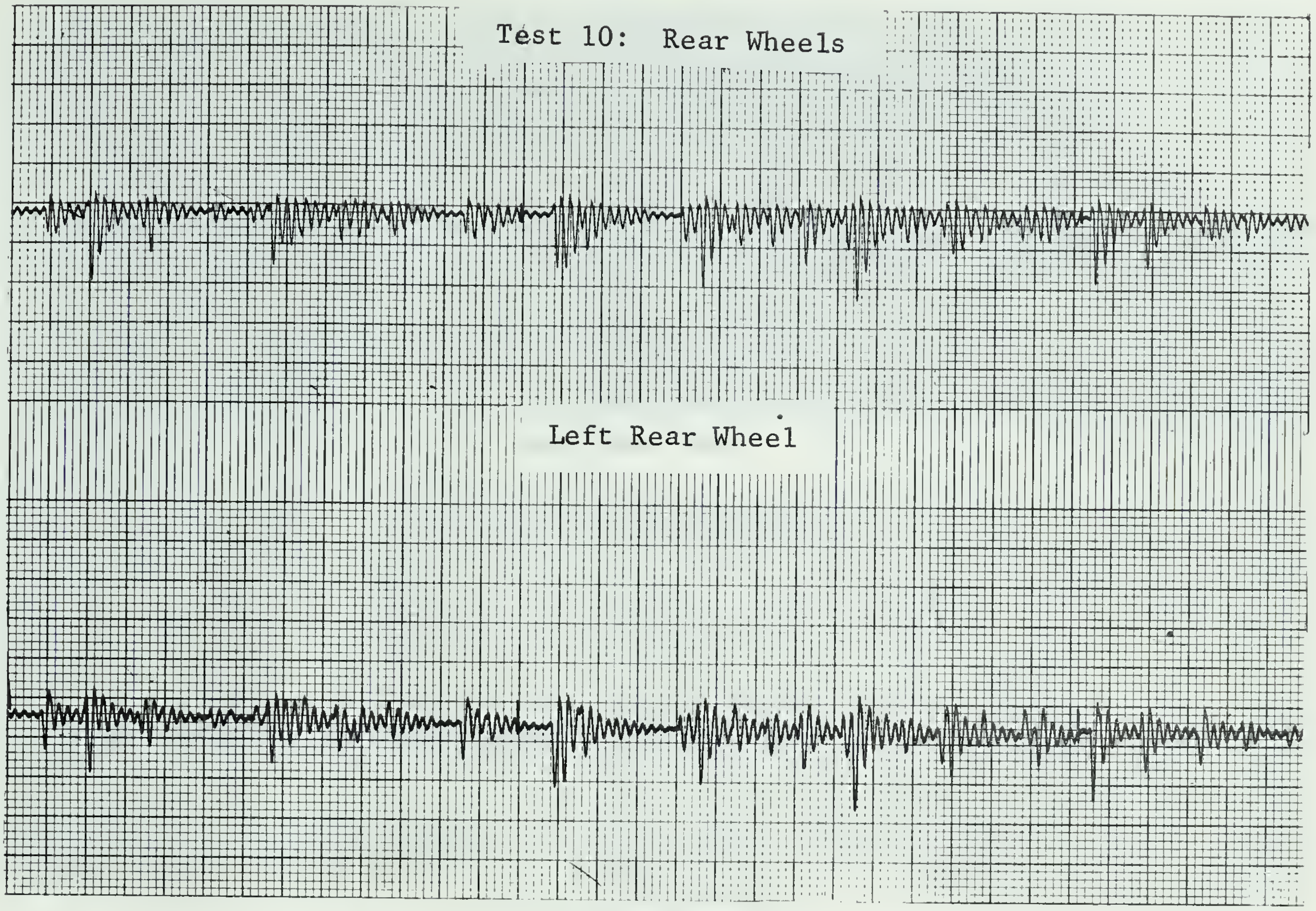
Right Rear Wheel



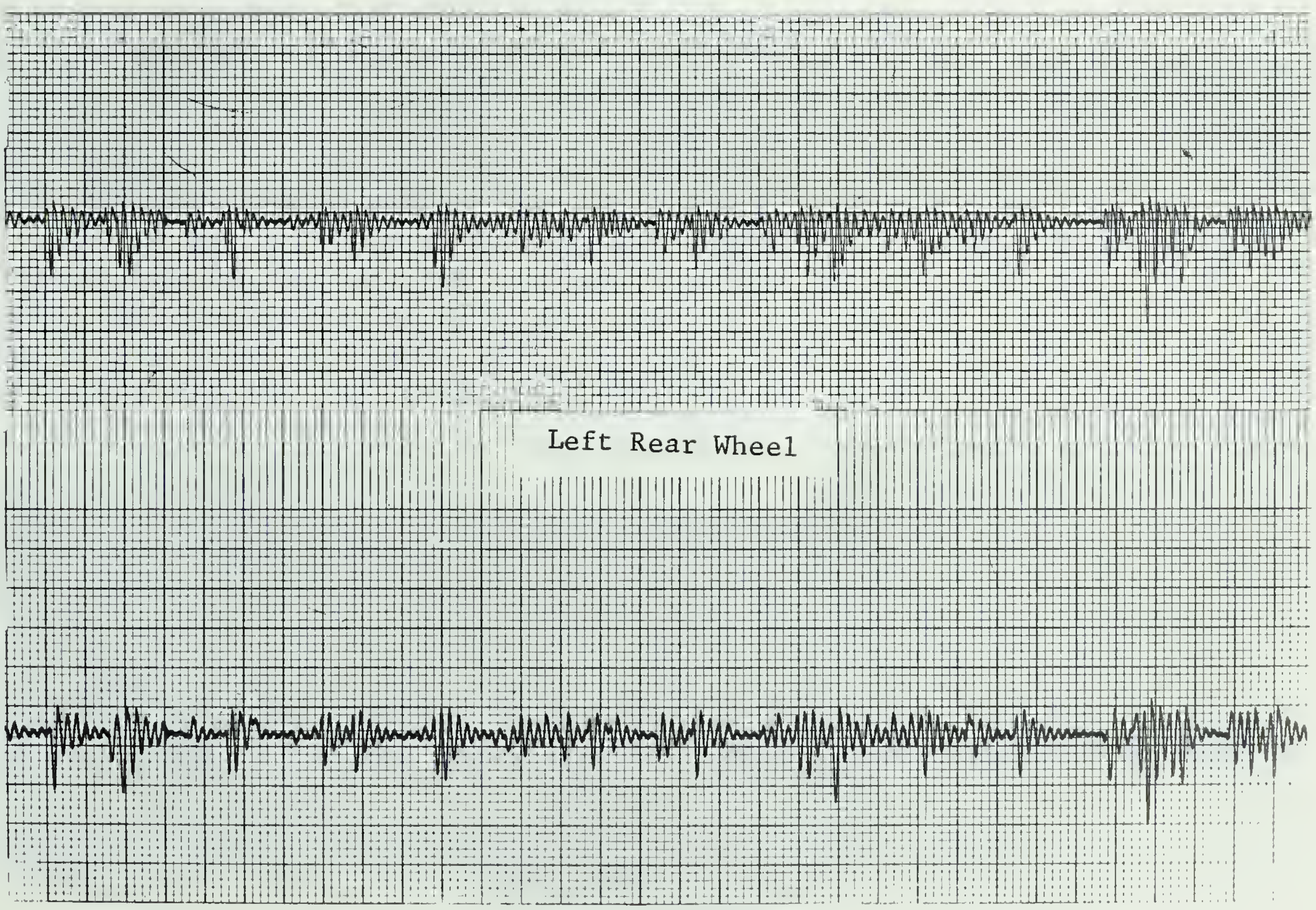
Right Rear Wheel



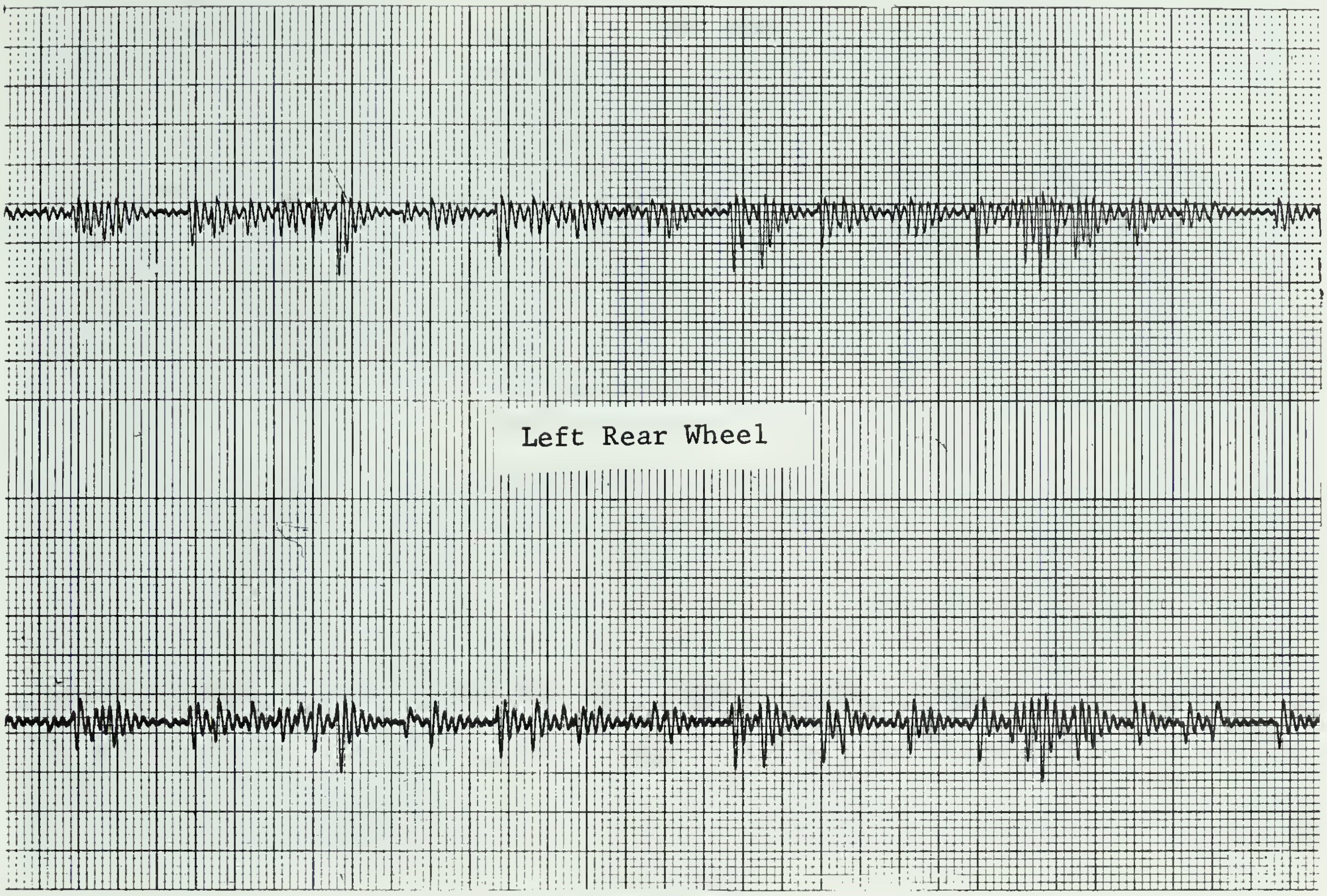
Right Rear Wheel



Right Rear Wheel



Right Rear Wheel



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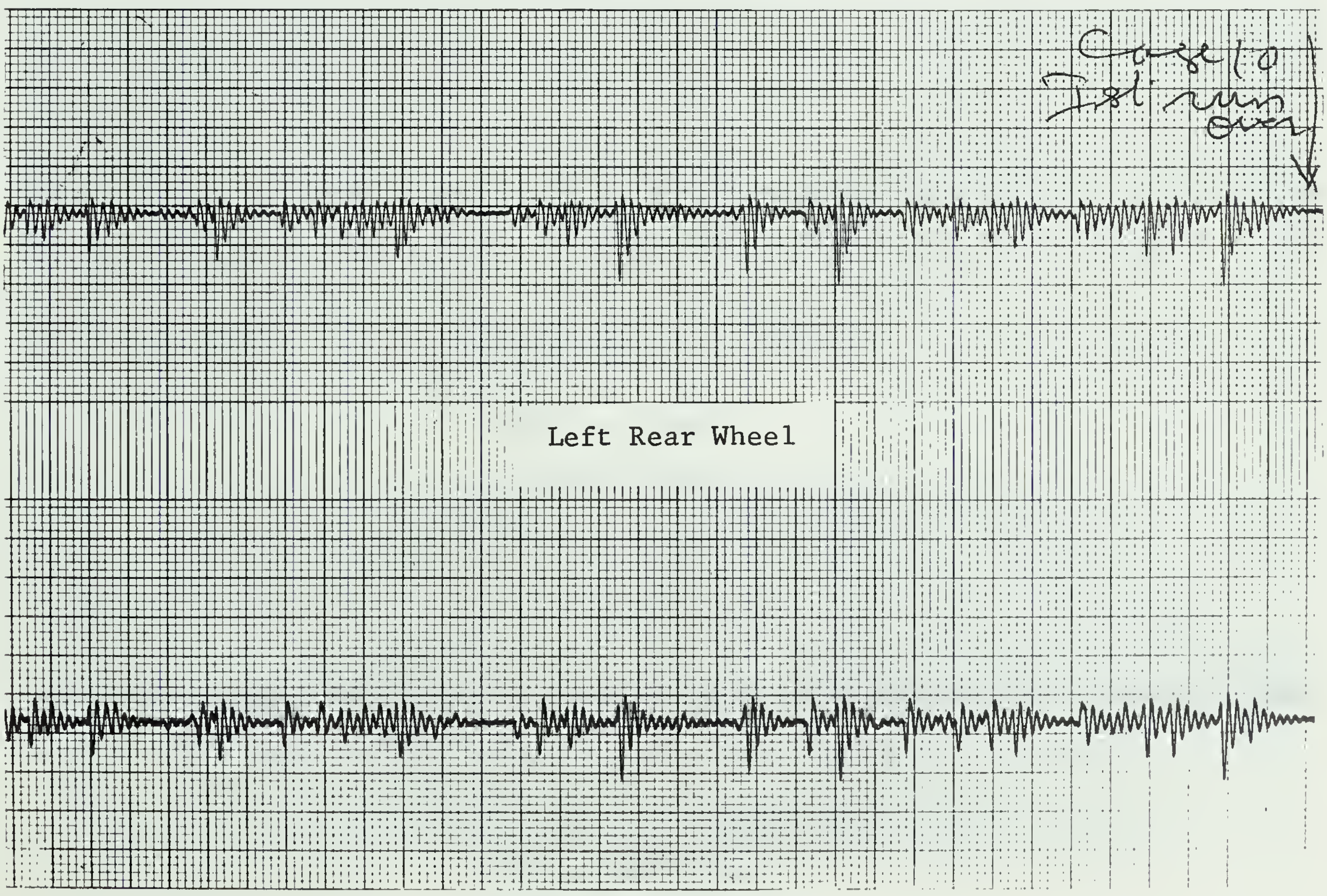
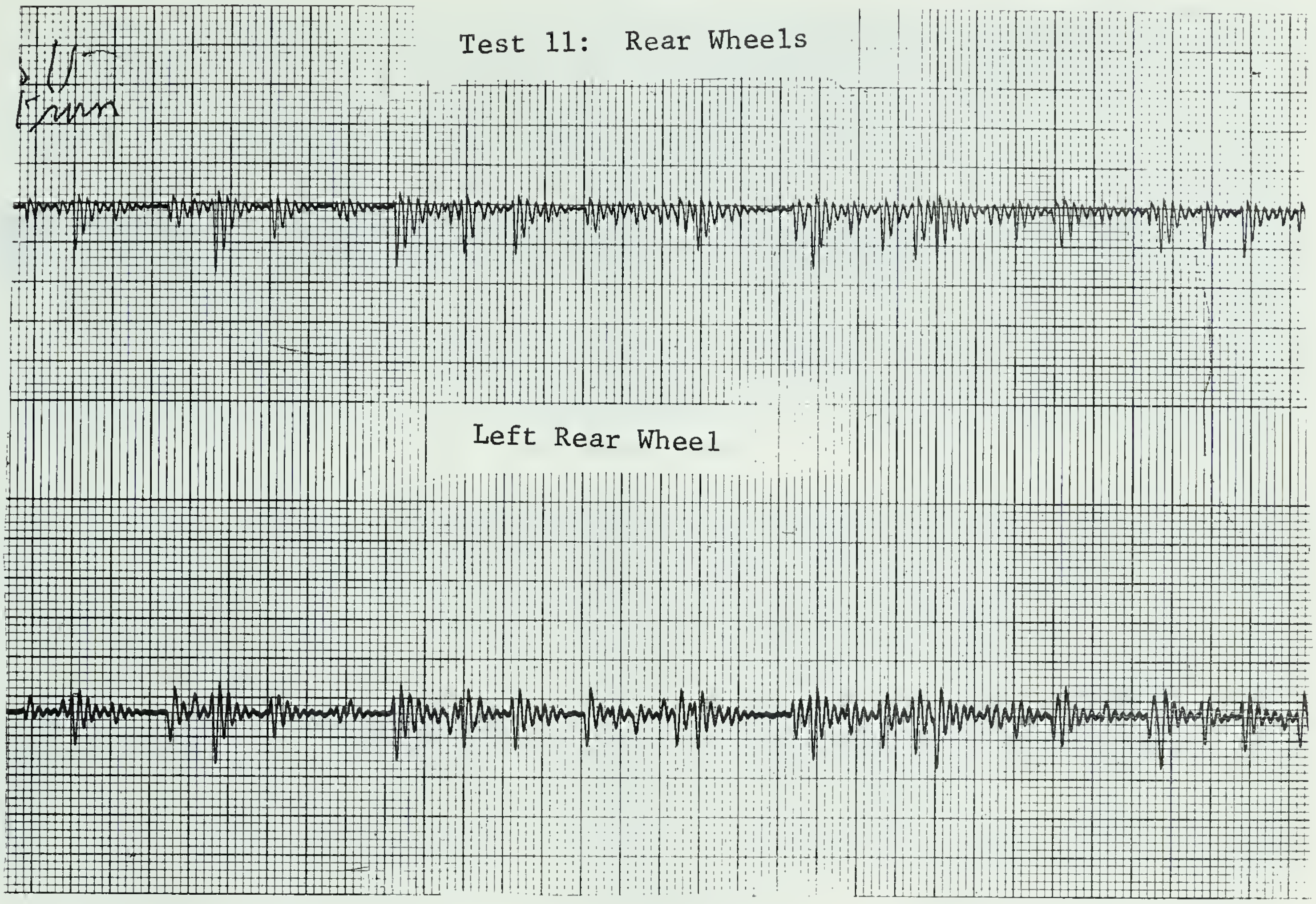


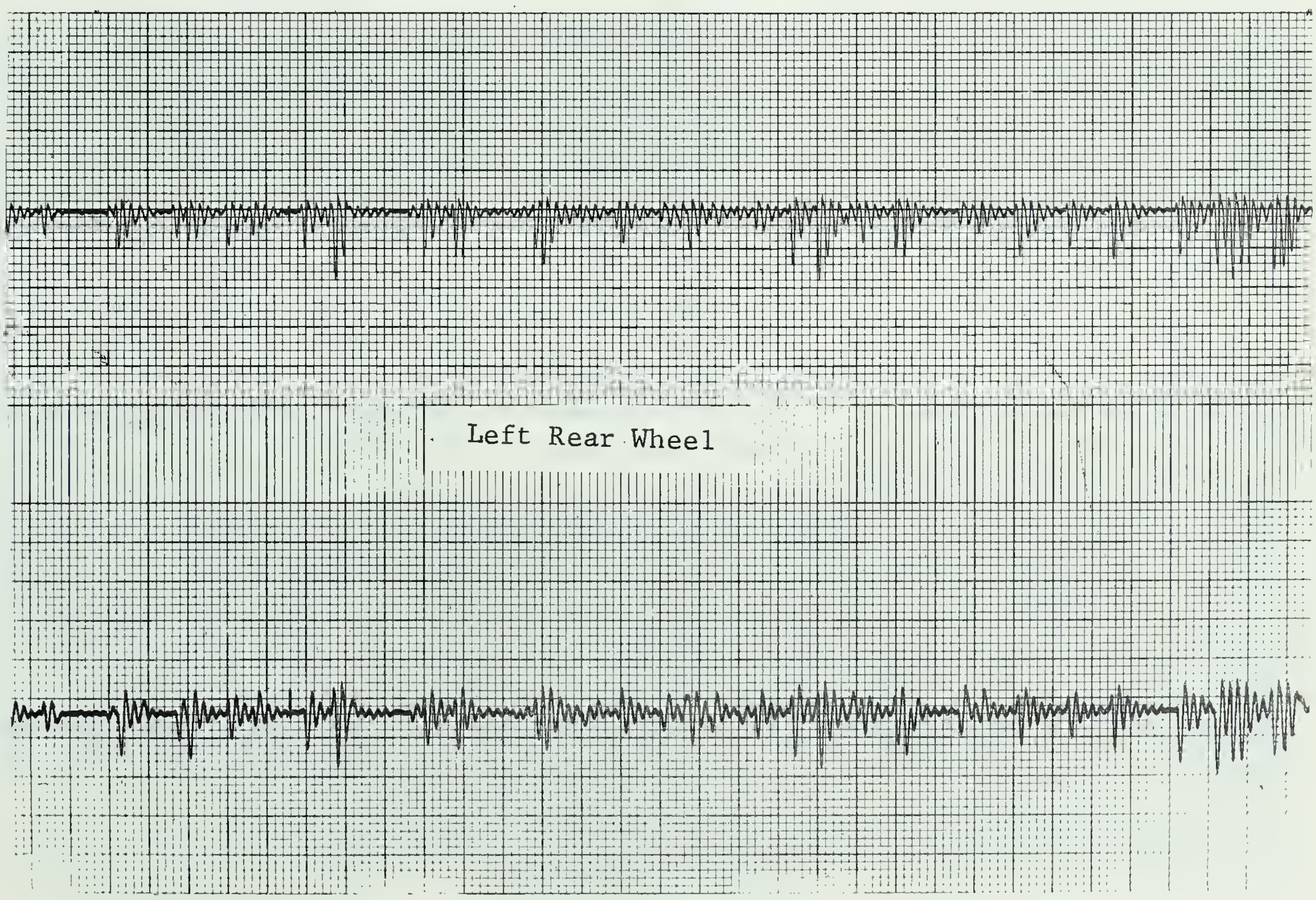
Right Rear Wheel

Test 11: Rear Wheels



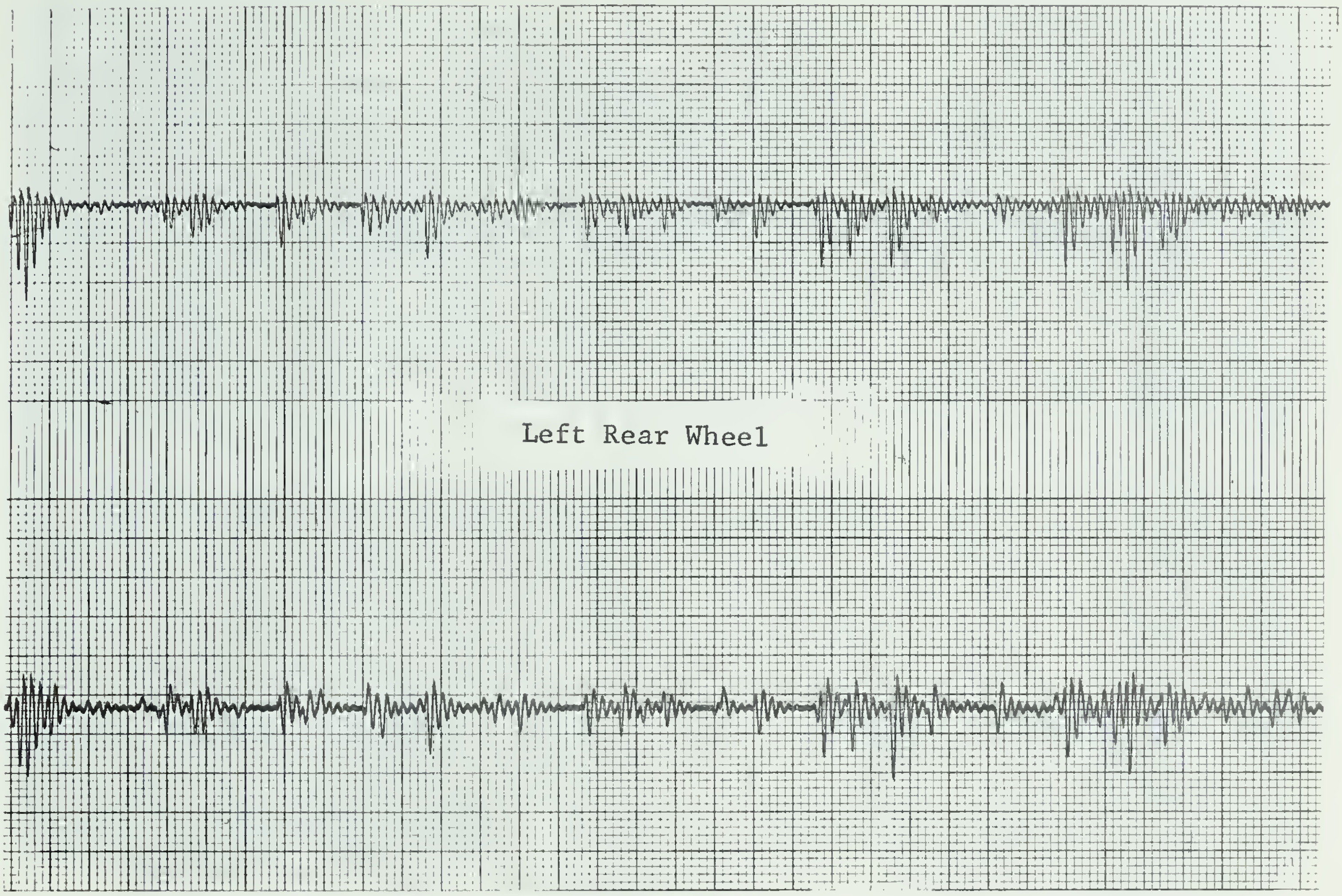
Left Rear Wheel

Right Rear Wheel

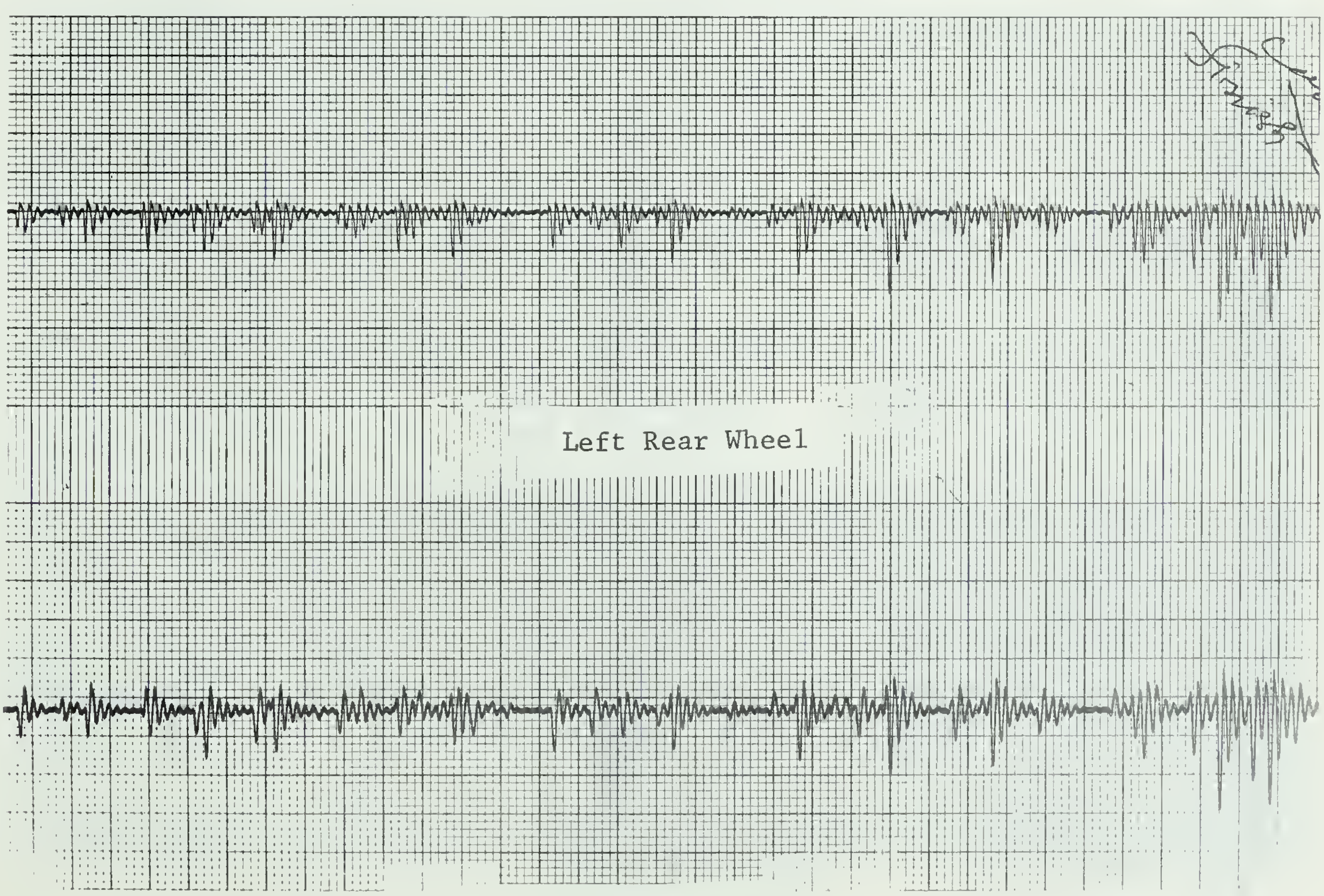


Left Rear Wheel

Right Rear Wheel

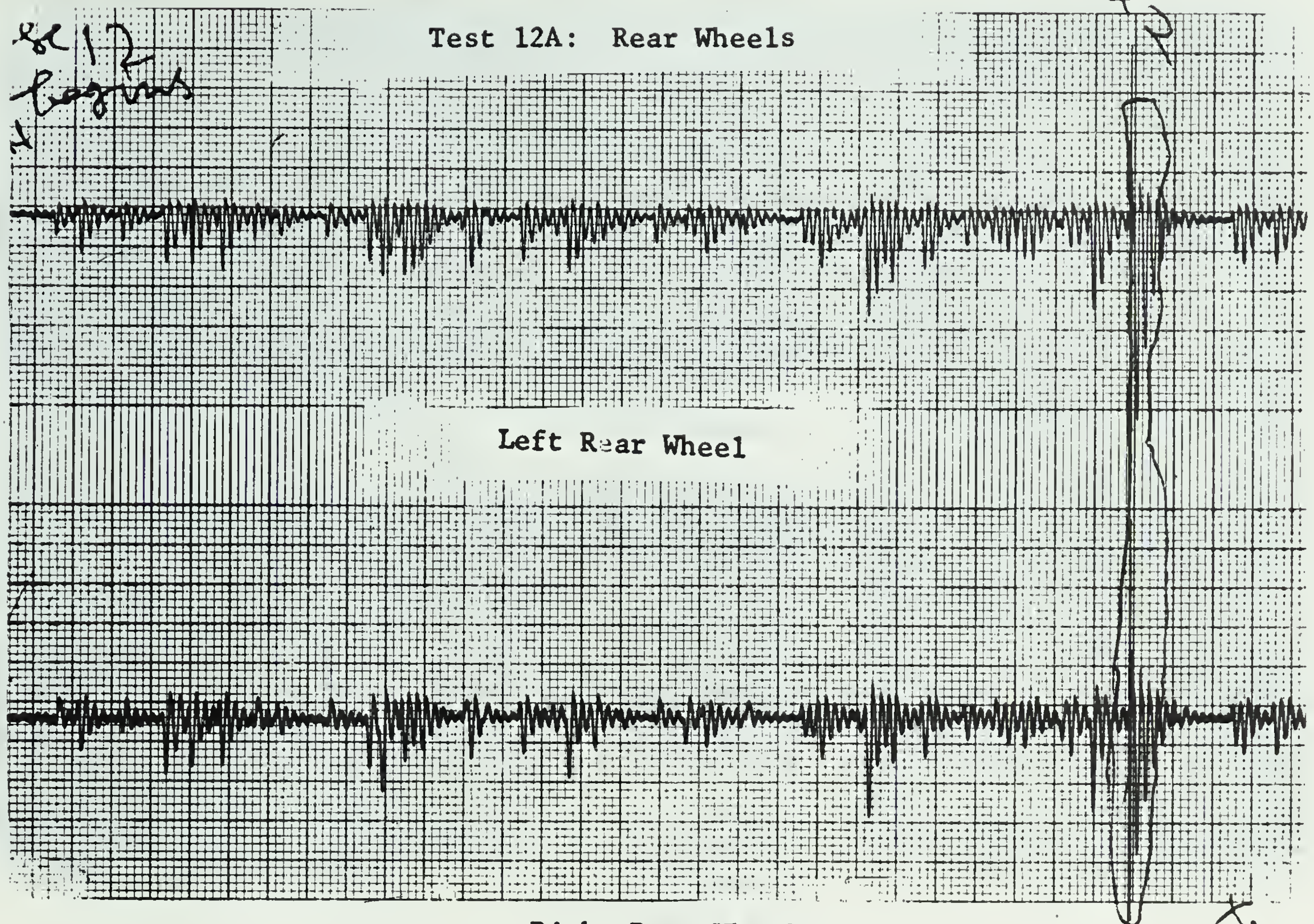


Right Rear Wheel

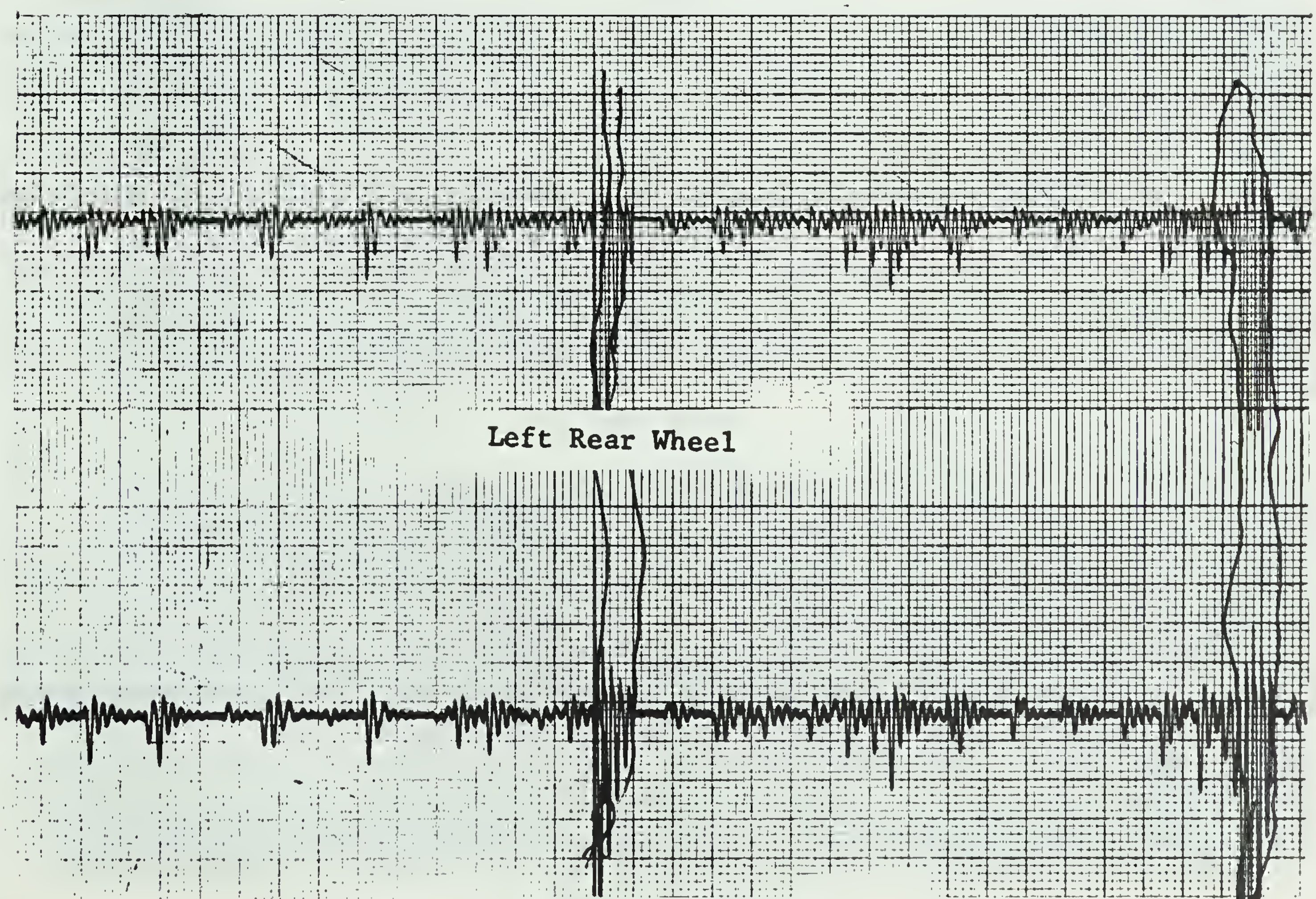


Right Rear Wheel

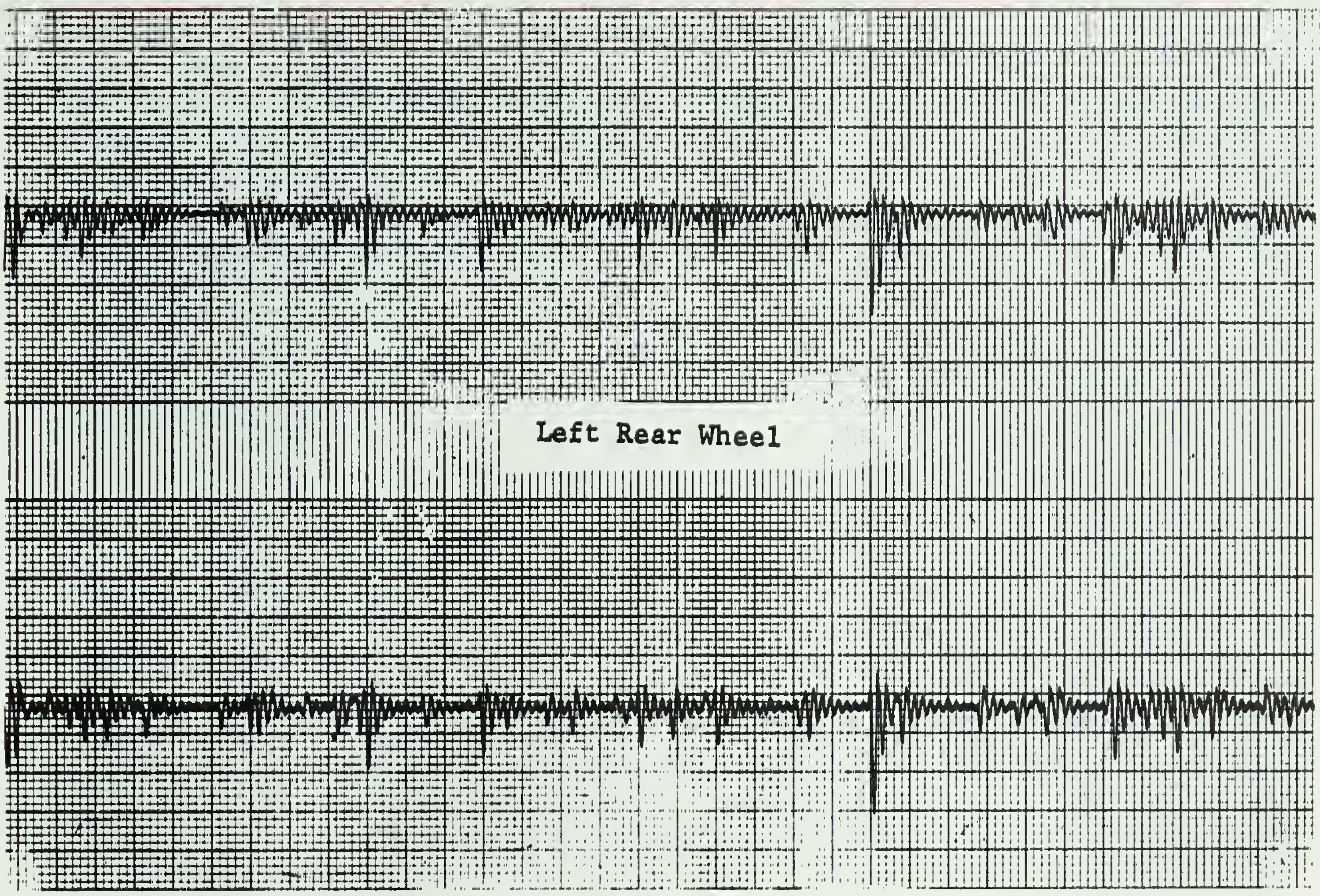
Test 12A: Rear Wheels



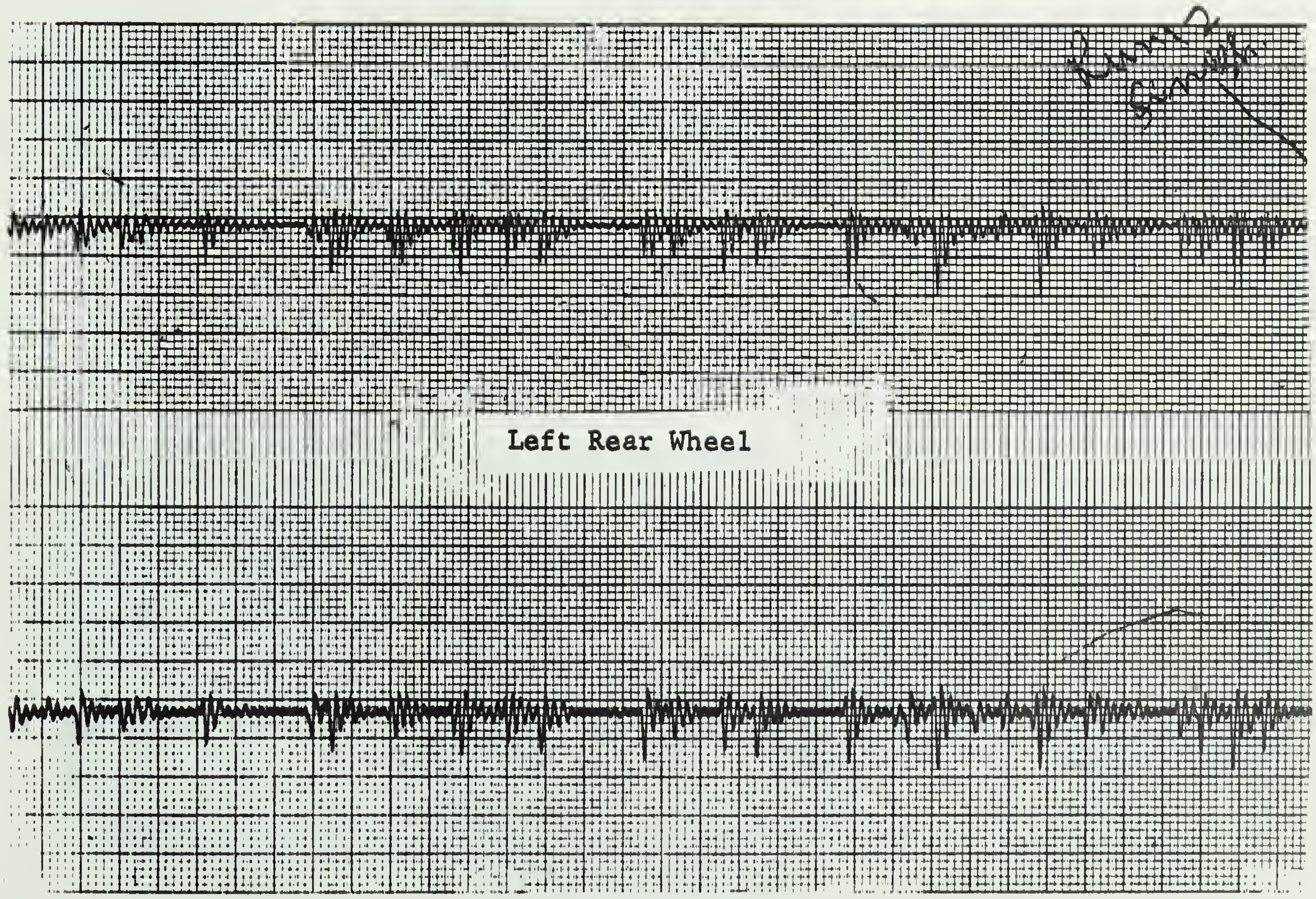
Right Rear Wheel



Right Rear Wheel

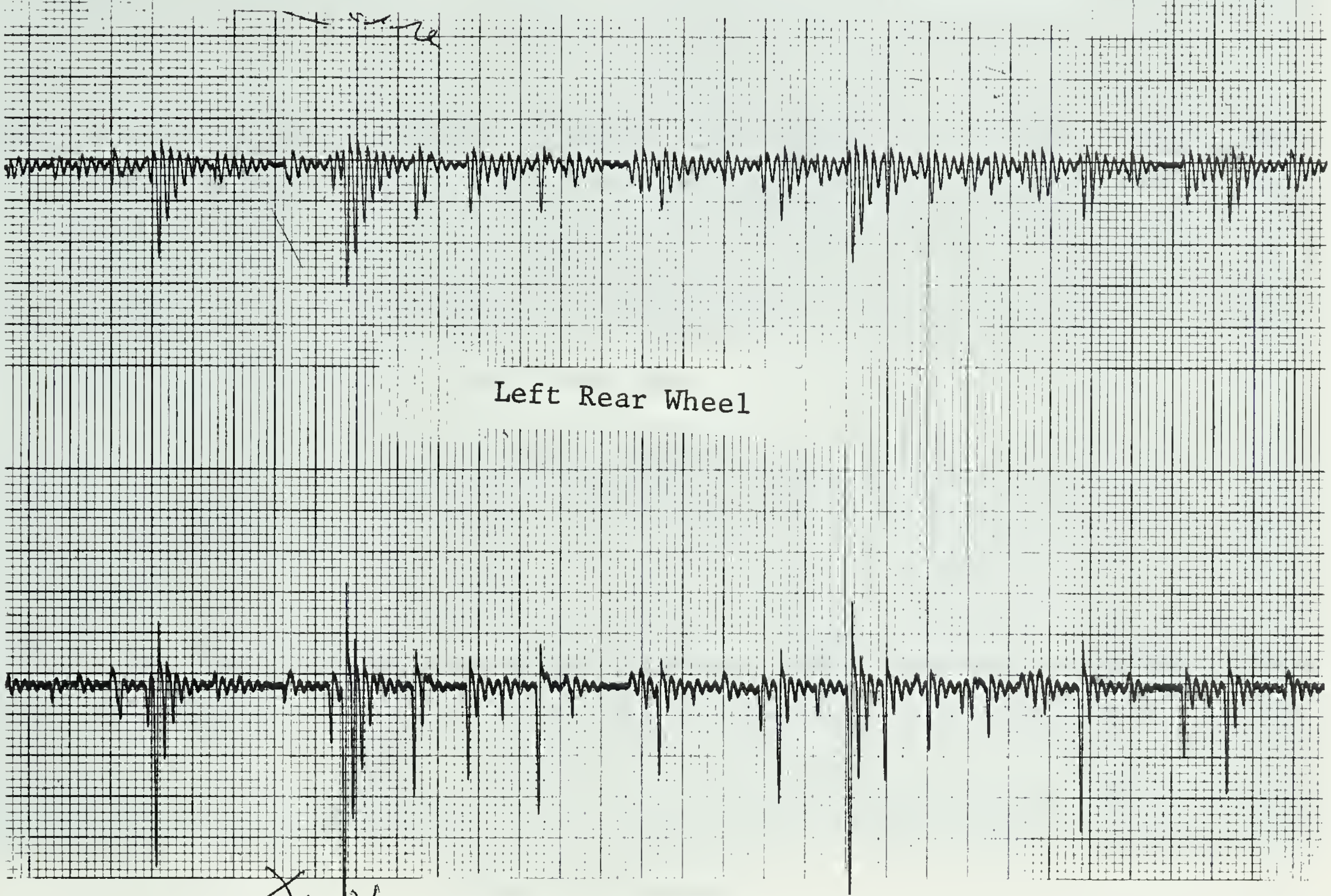


Right Rear Wheel



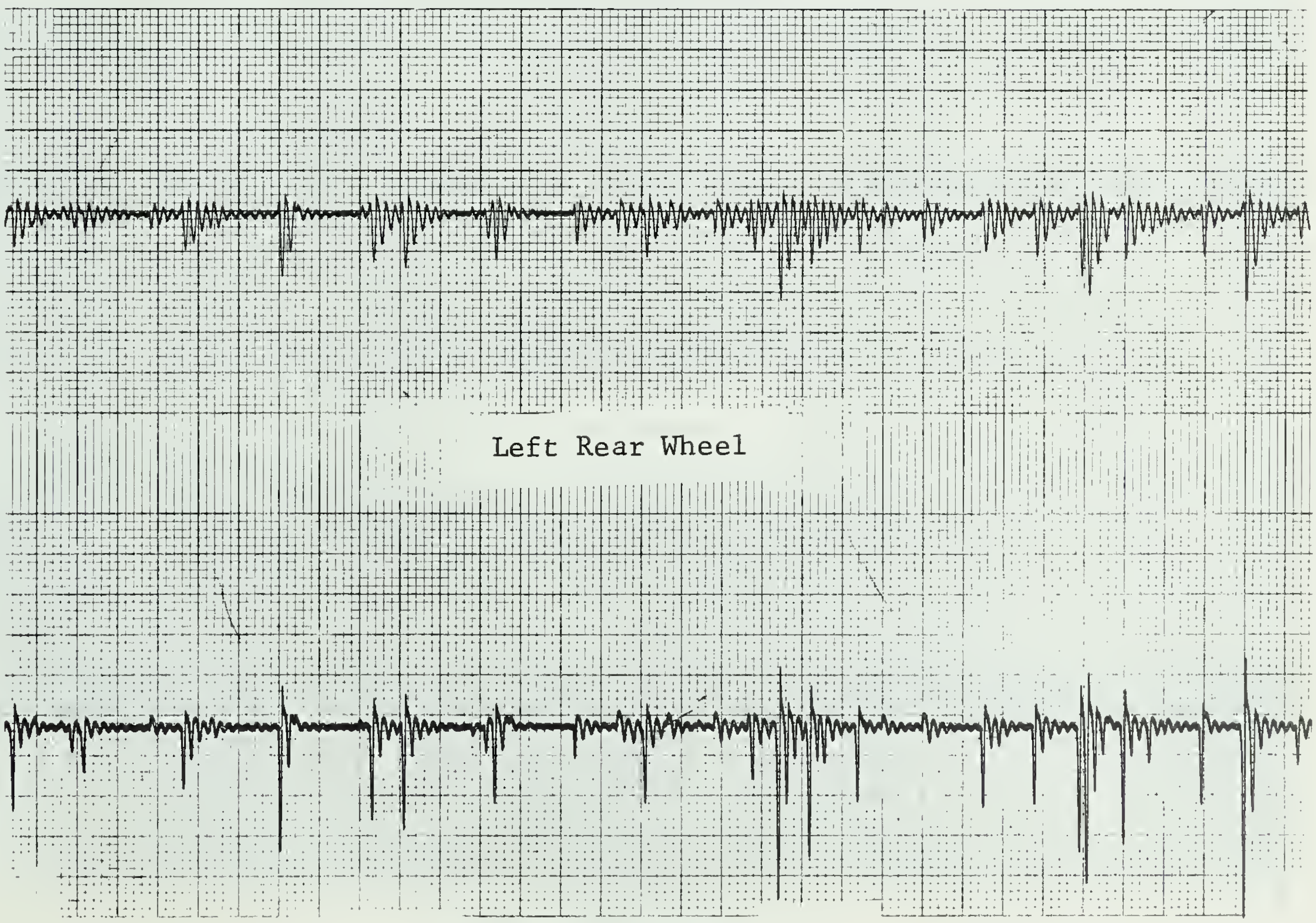
Right Rear Wheel

Test 12B: Left Rear Wheel and Front Axle Centre

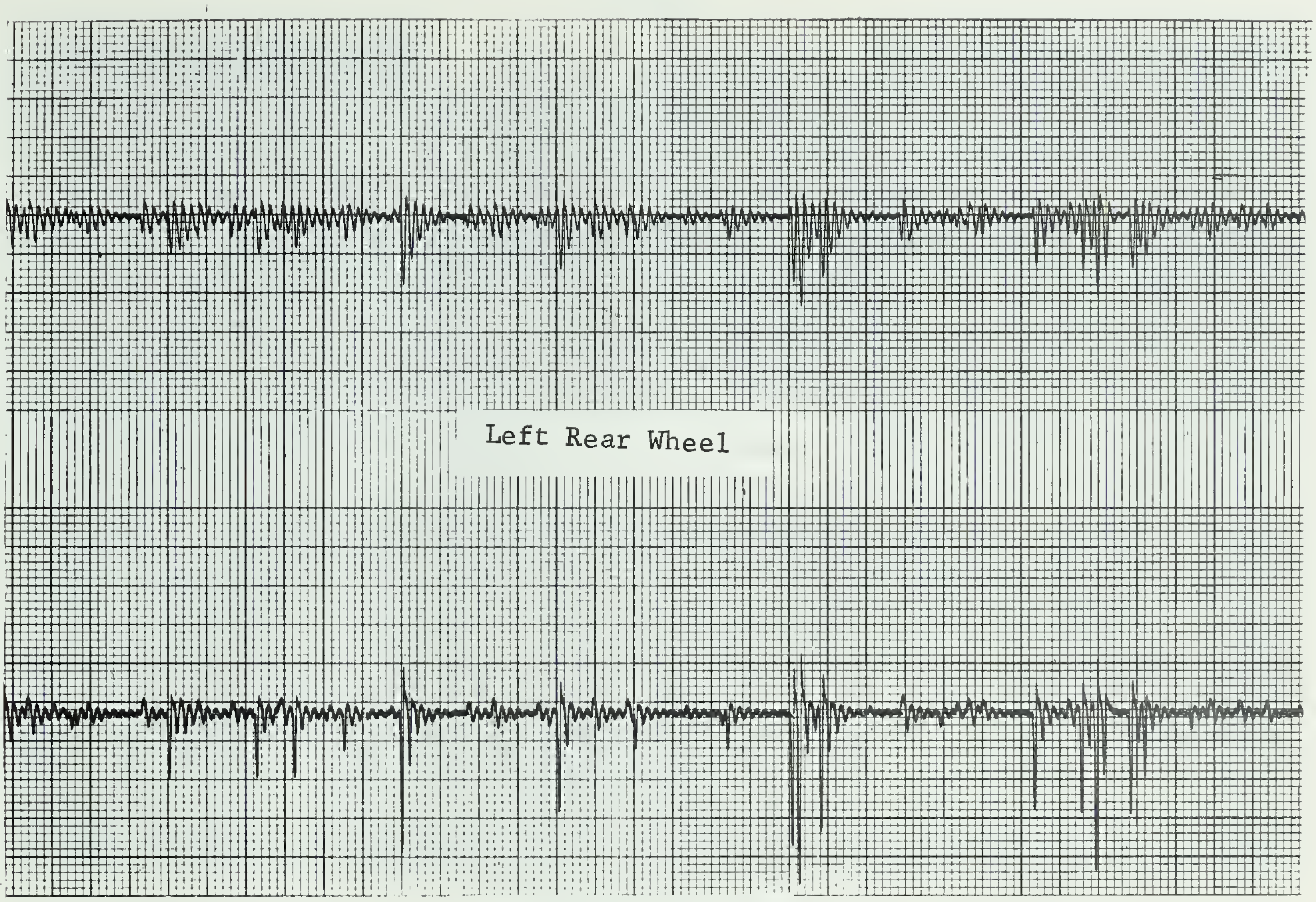


X1 X5

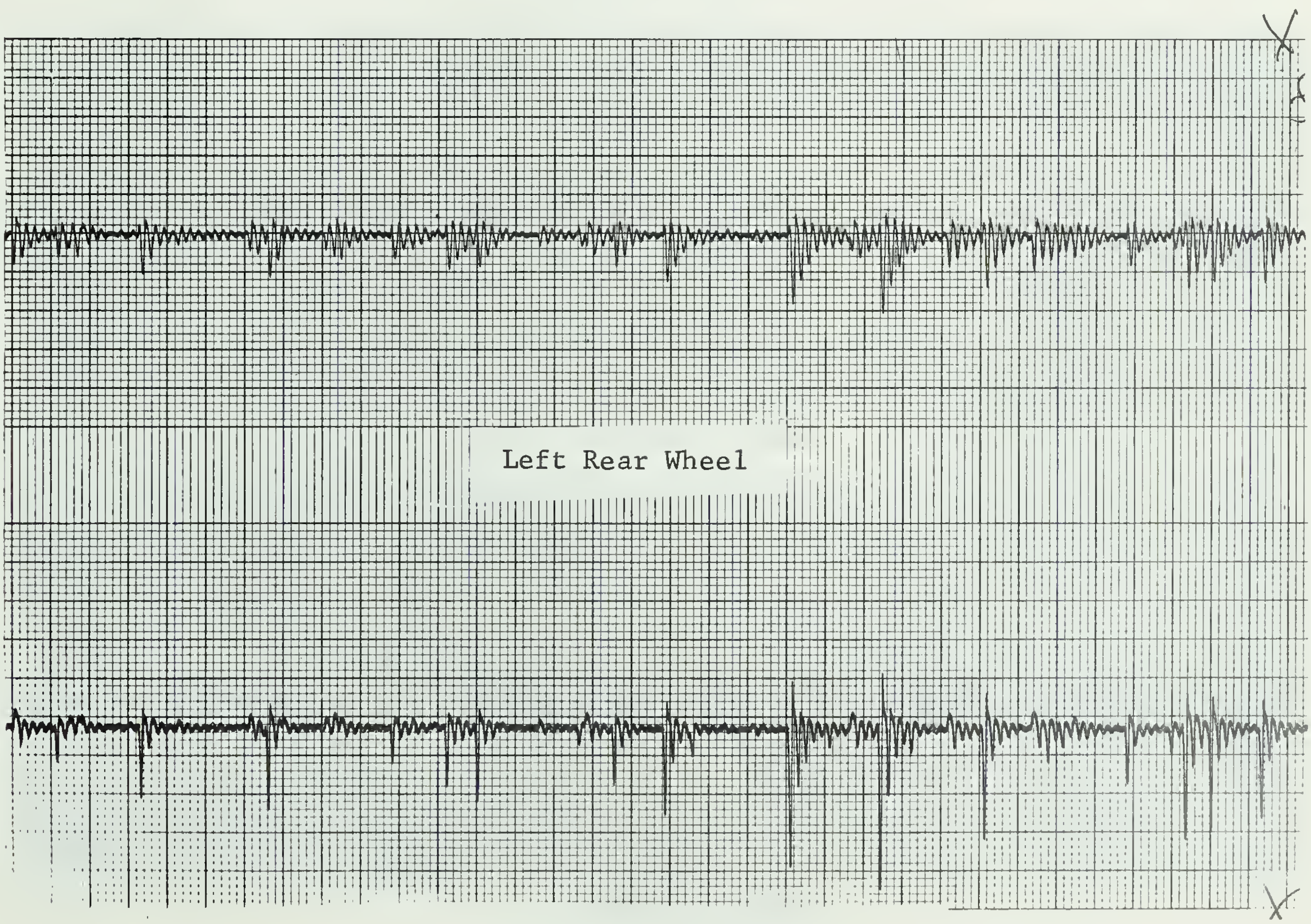
Front Axle Centre



Front Axle Centre

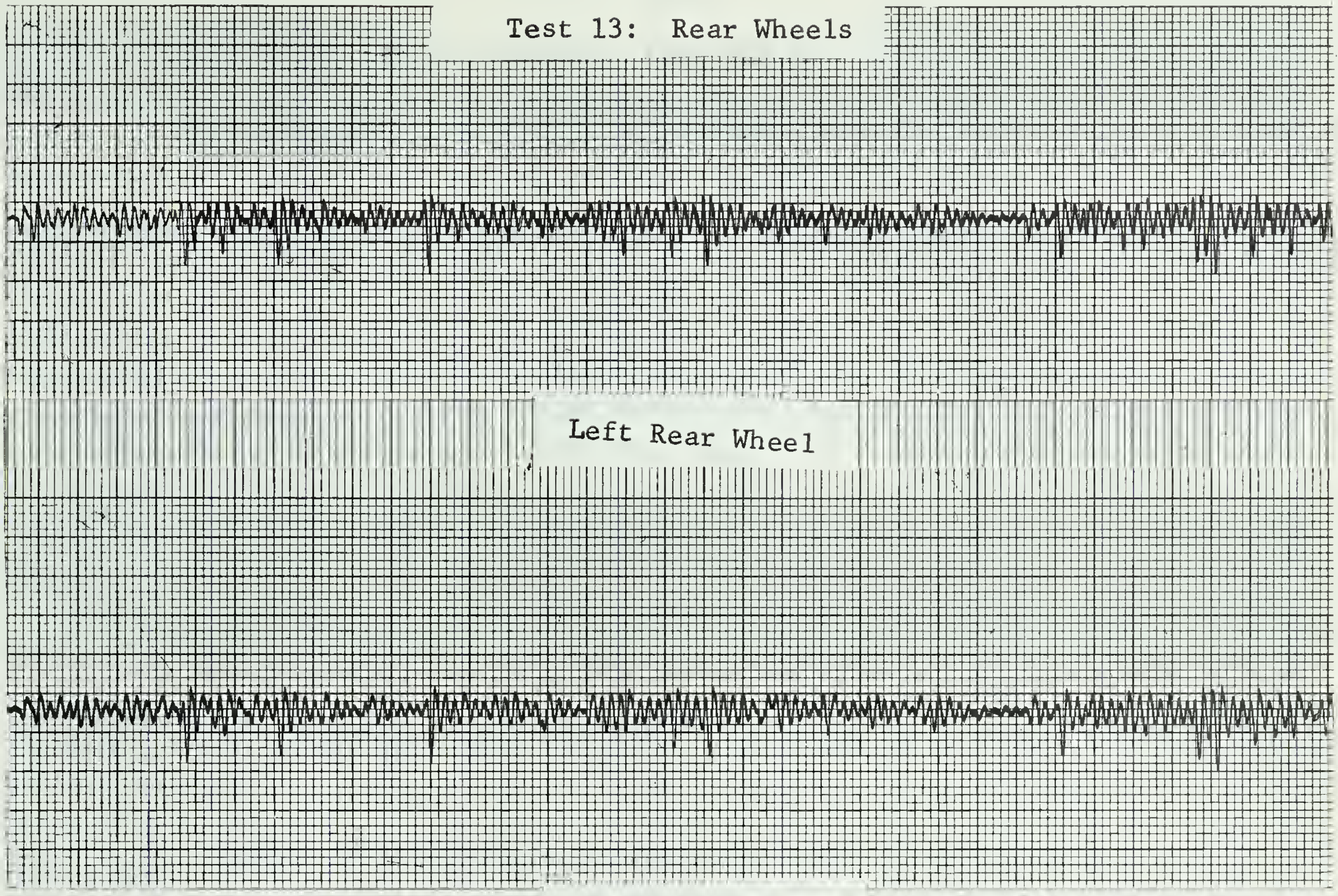


Front Axle Centre

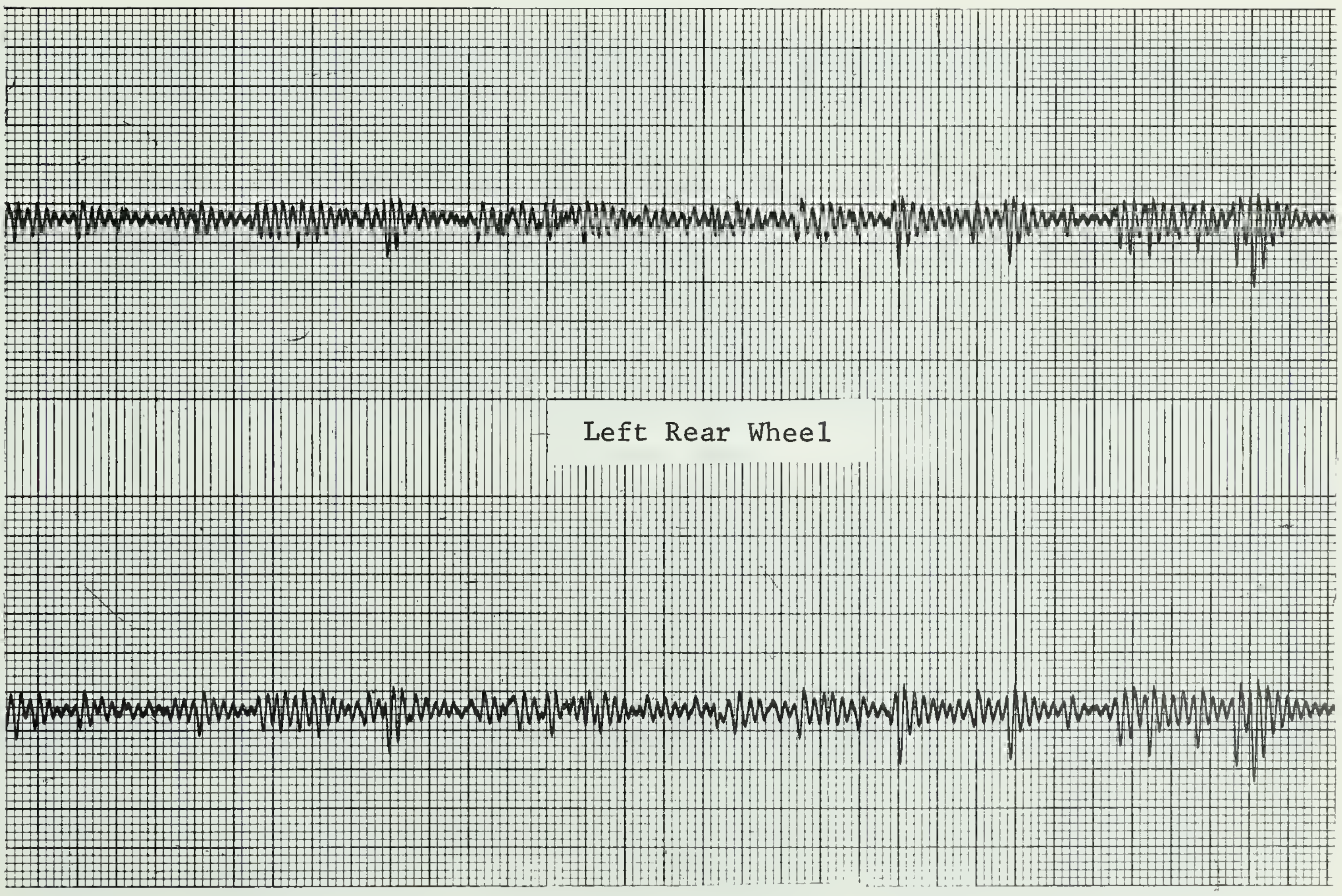


Front Axle Centre

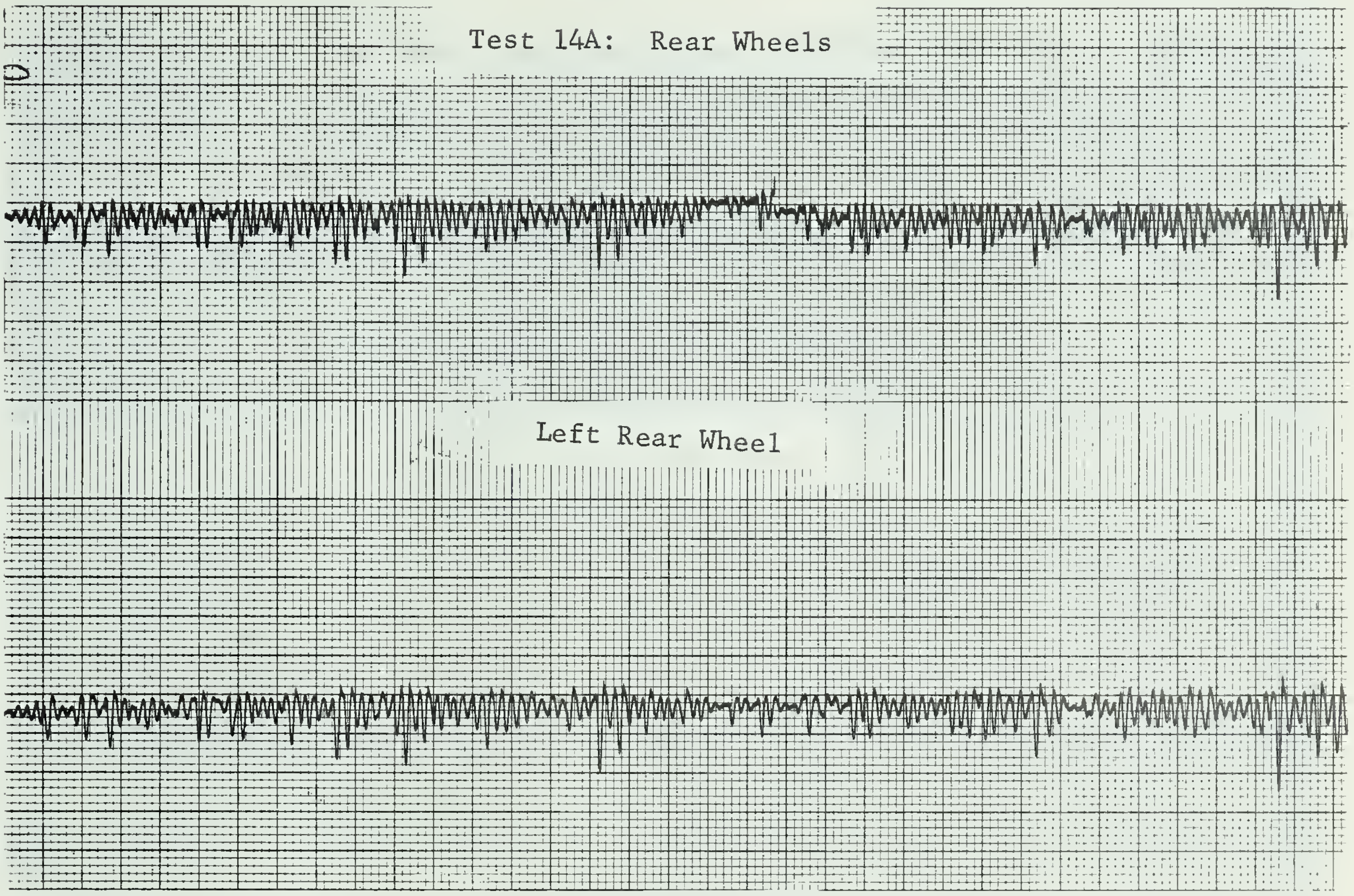
Test 13: Rear Wheels



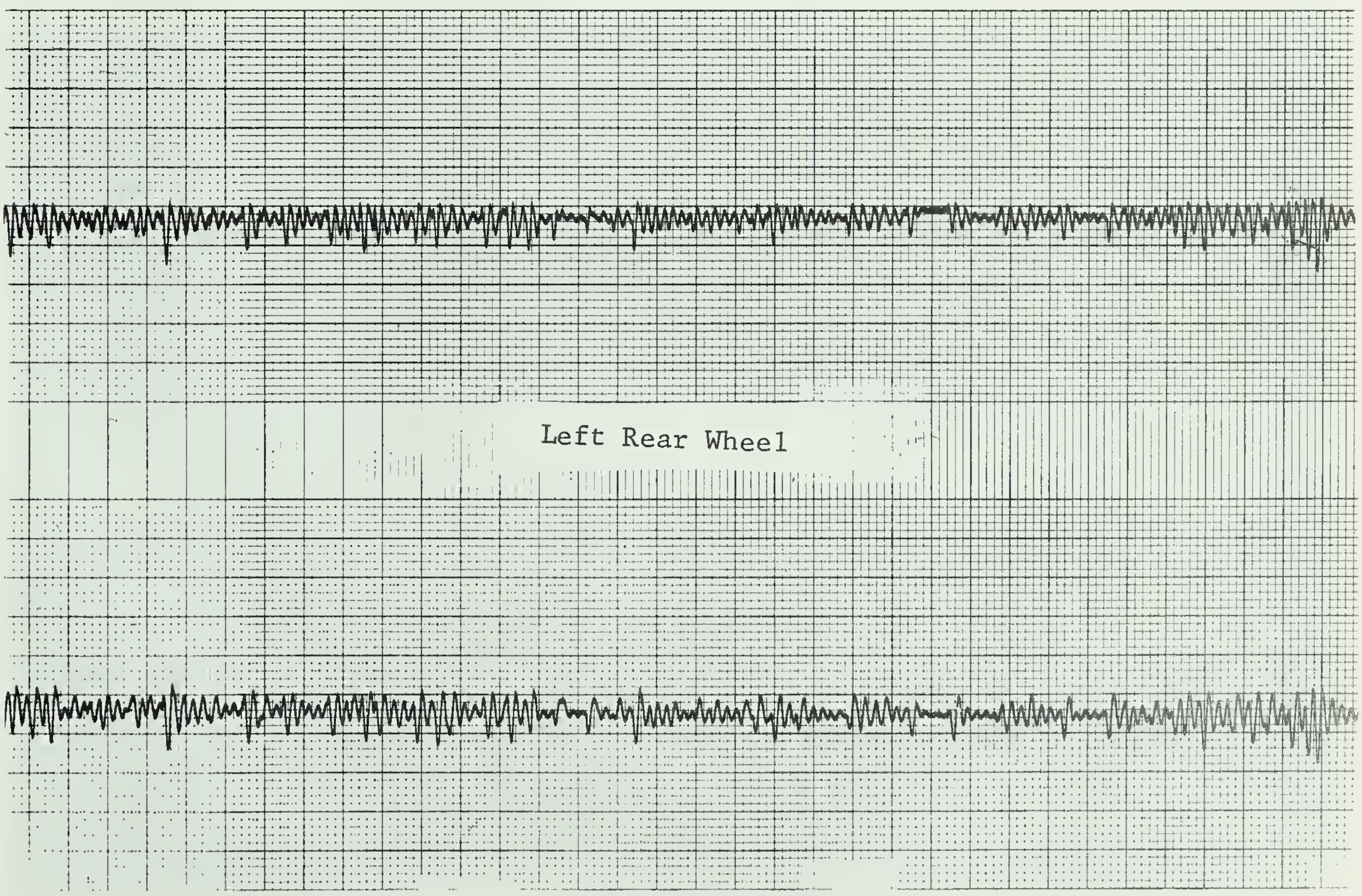
Right Rear Wheel



Right Rear Wheel

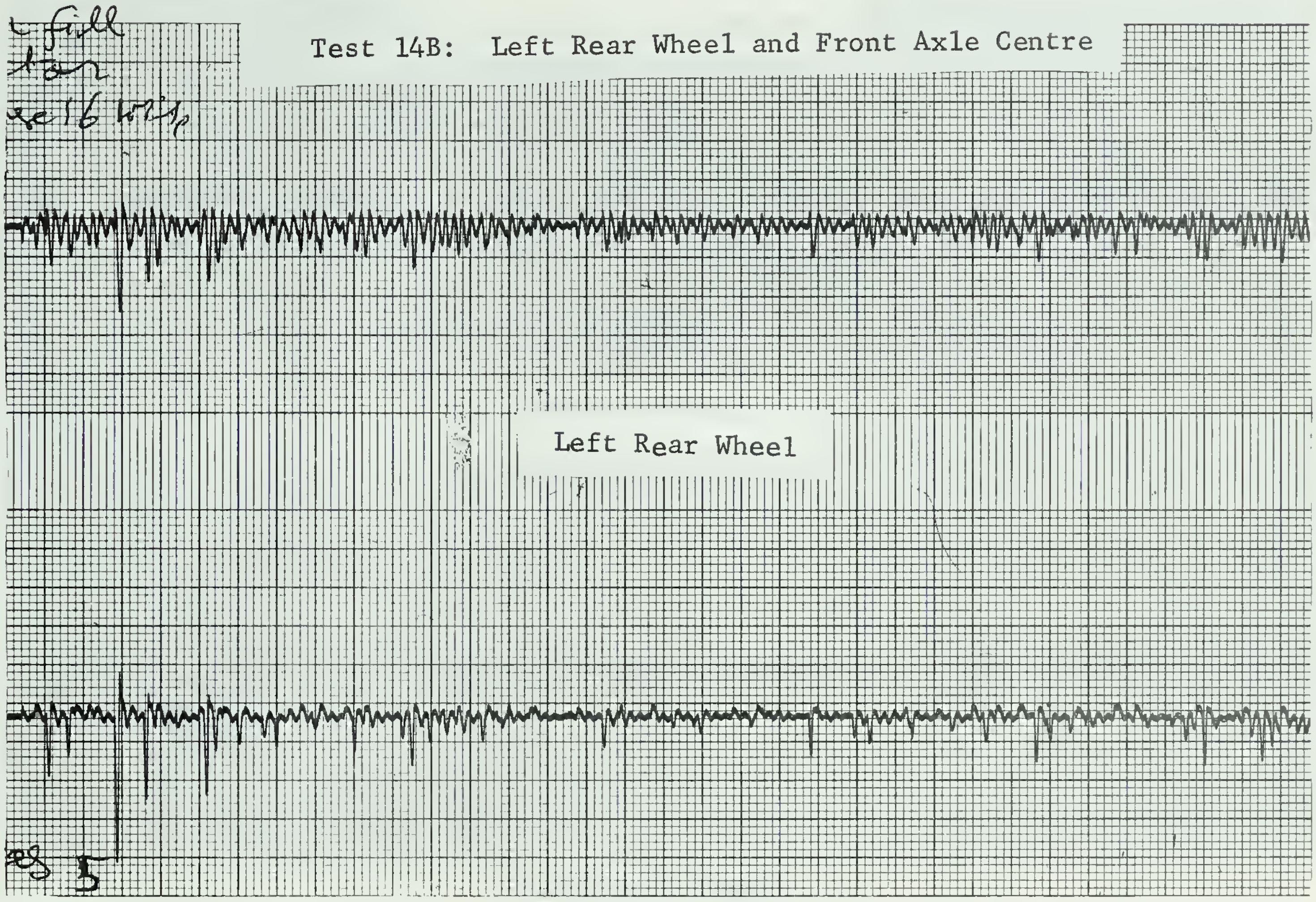


Right Rear Wheel

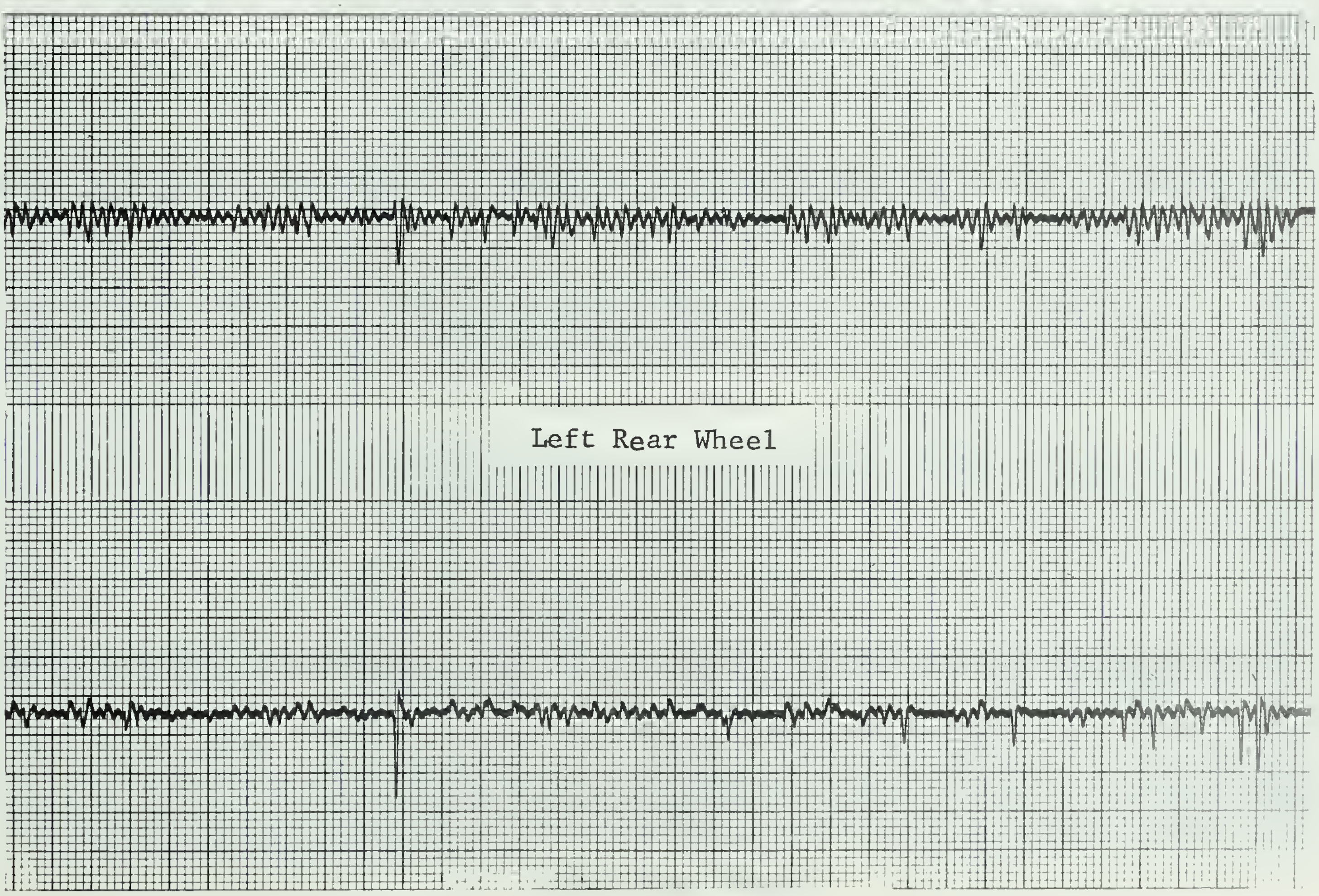


Right Rear Wheel

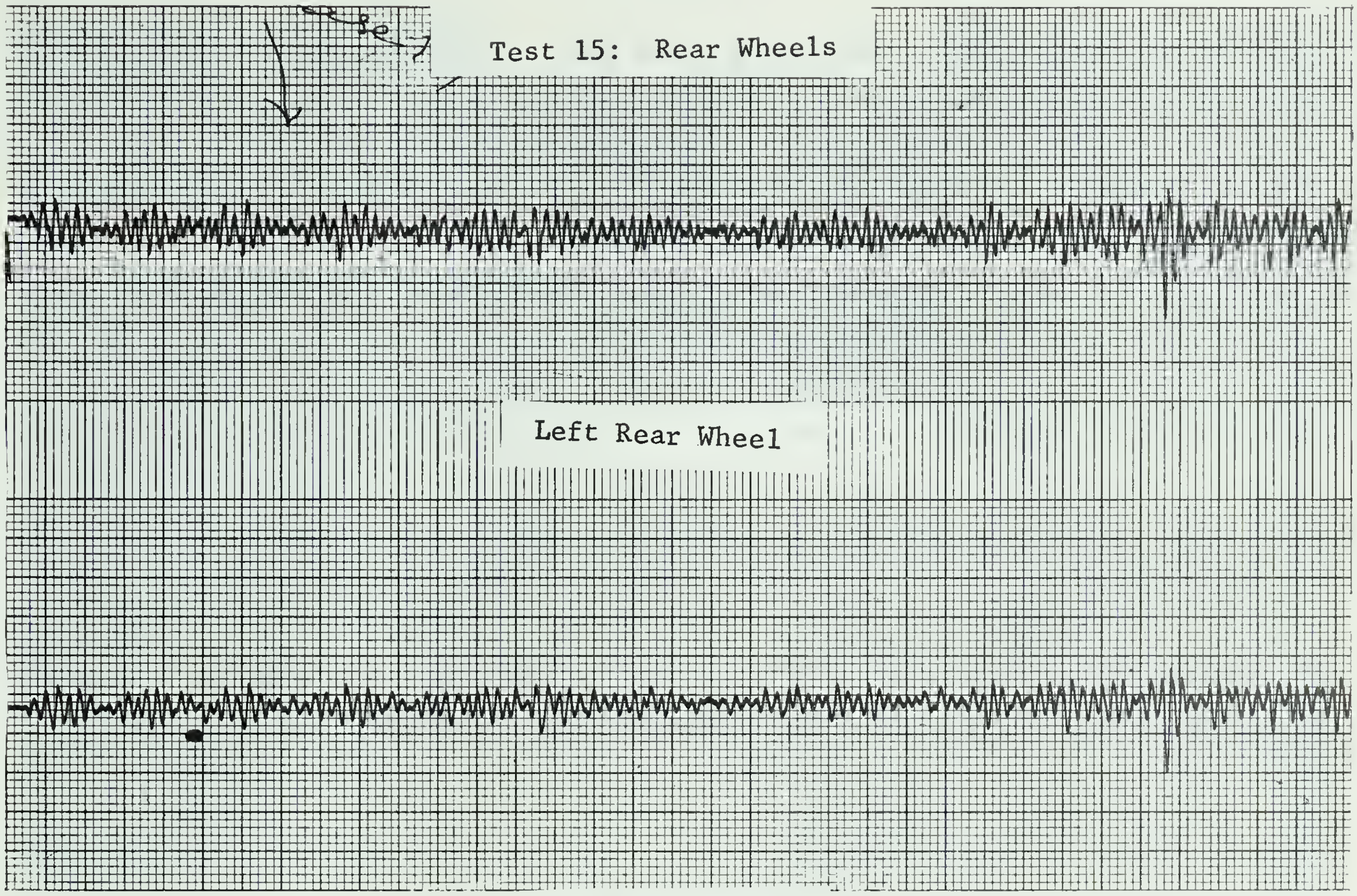
Test 14B: Left Rear Wheel and Front Axle Centre



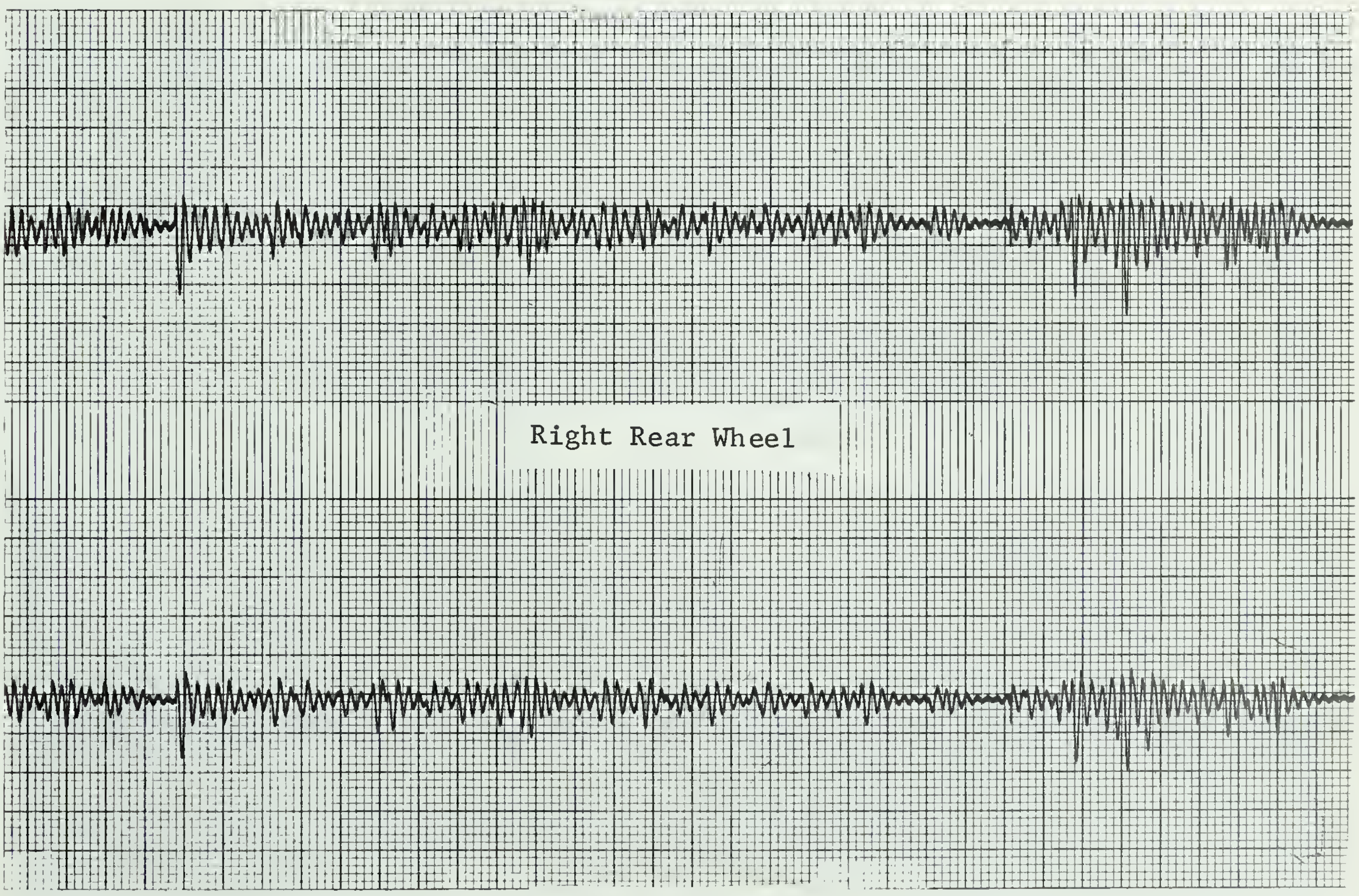
Front Axle Centre



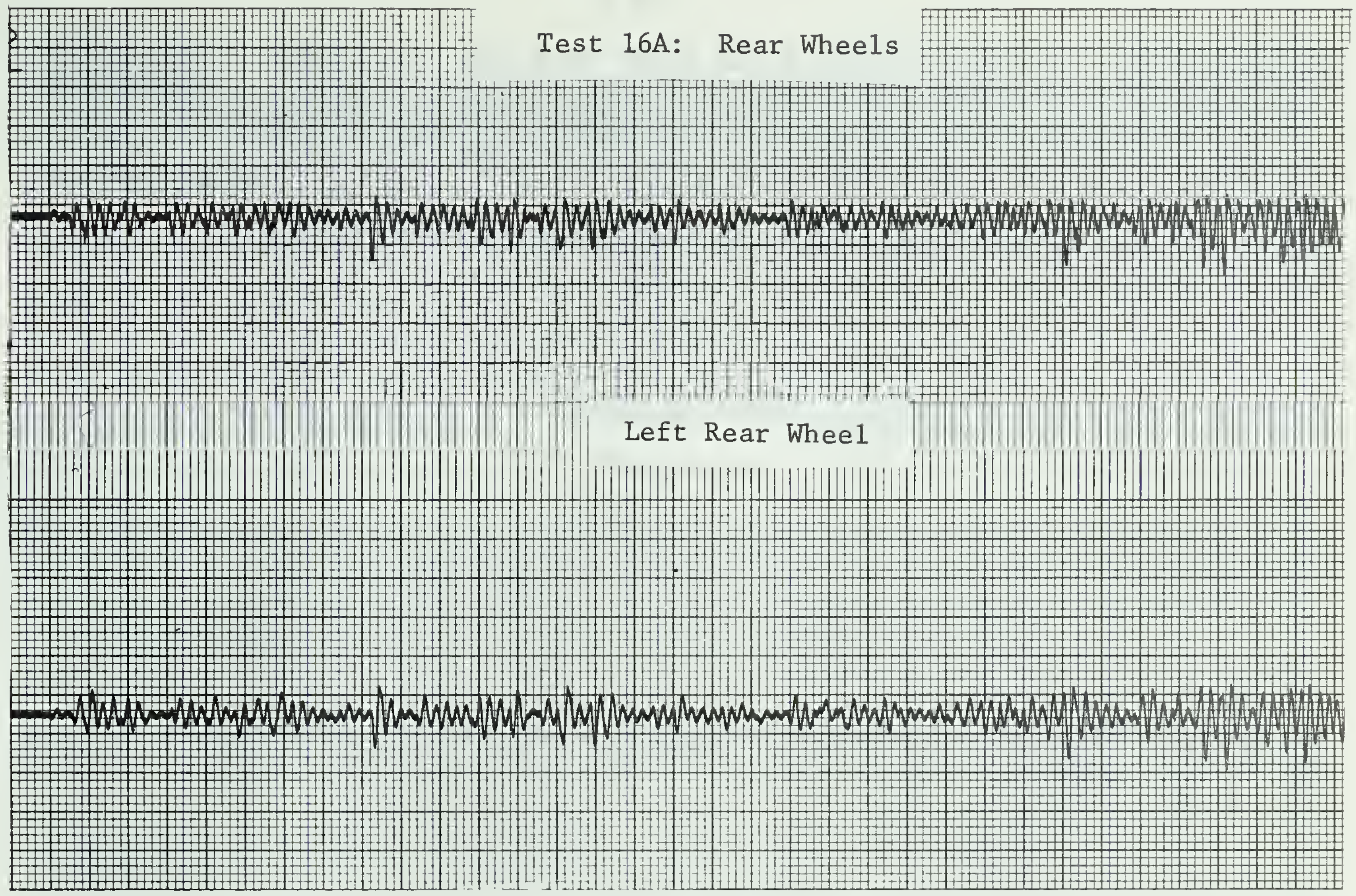
Front Axle Centre



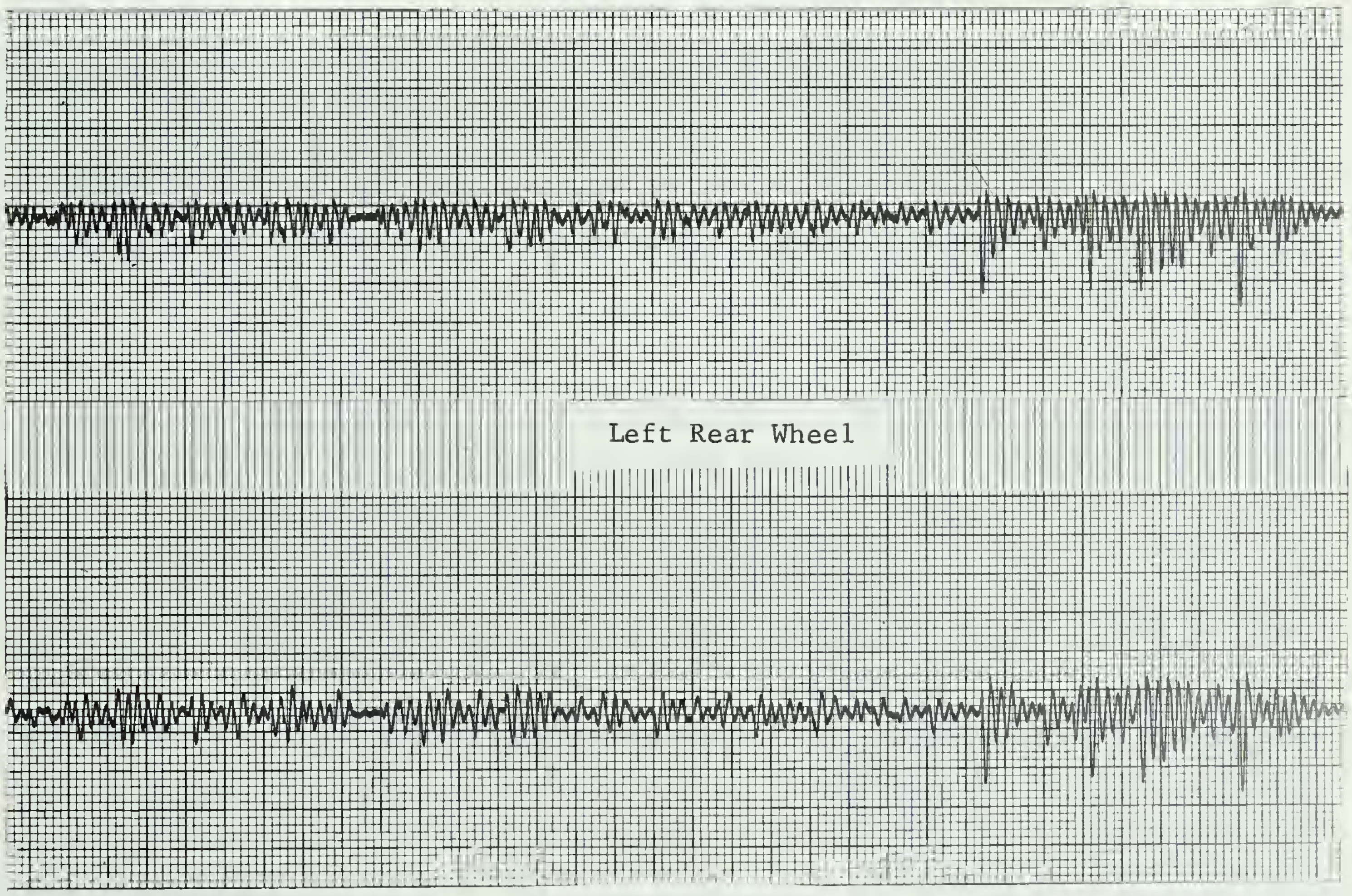
Right Rear Wheel



Left Rear Wheel

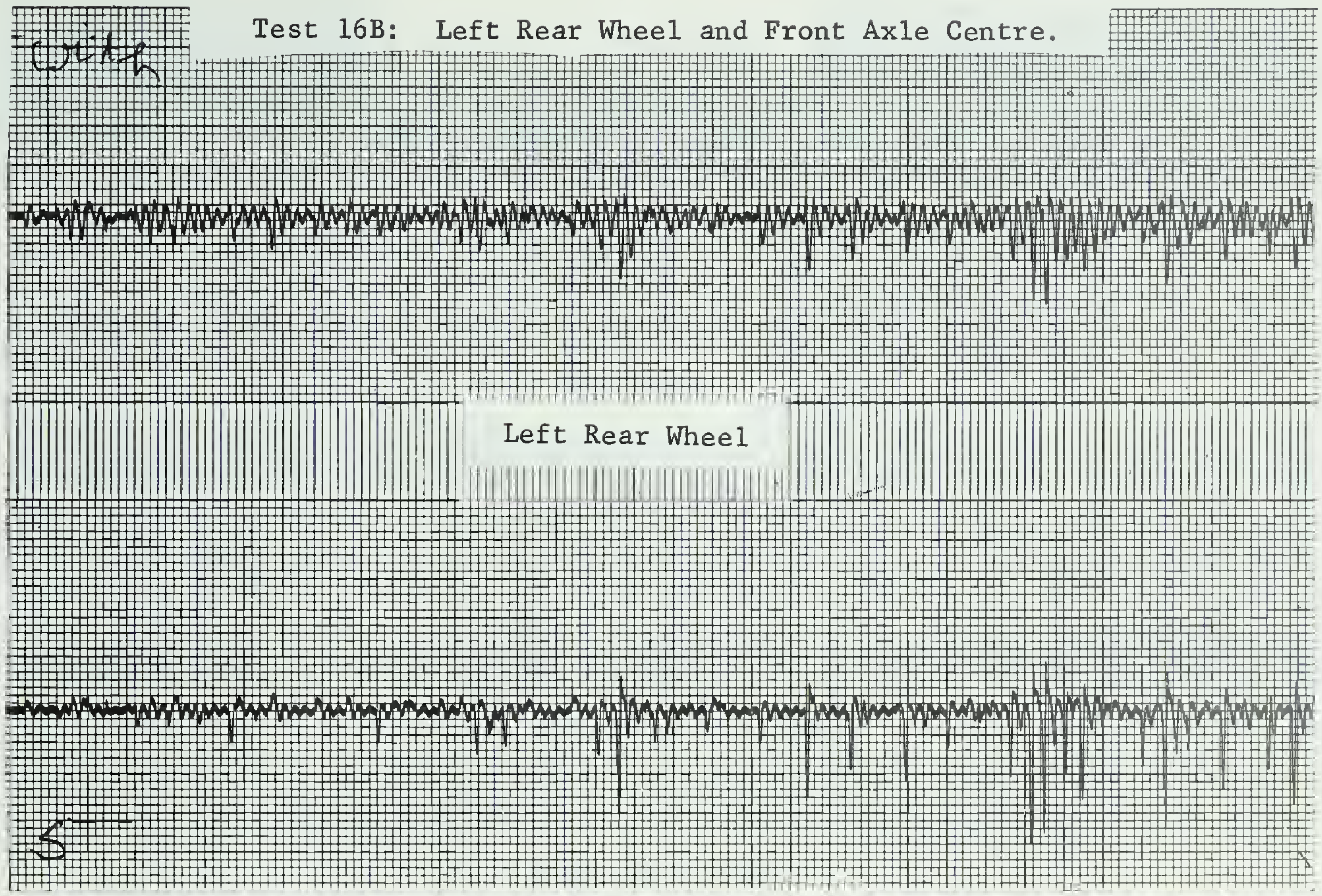


Right Rear Wheel

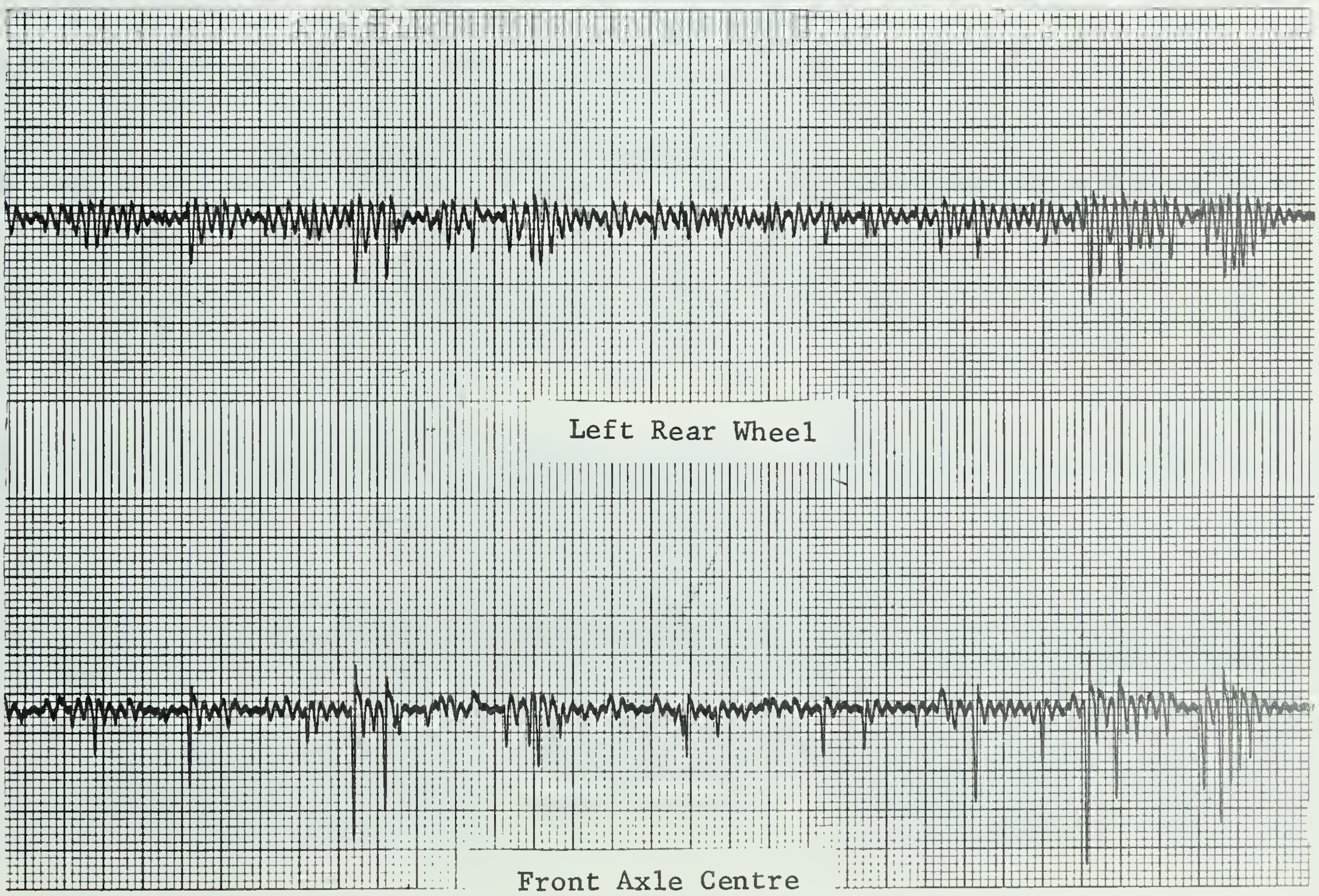


Right Rear Wheel

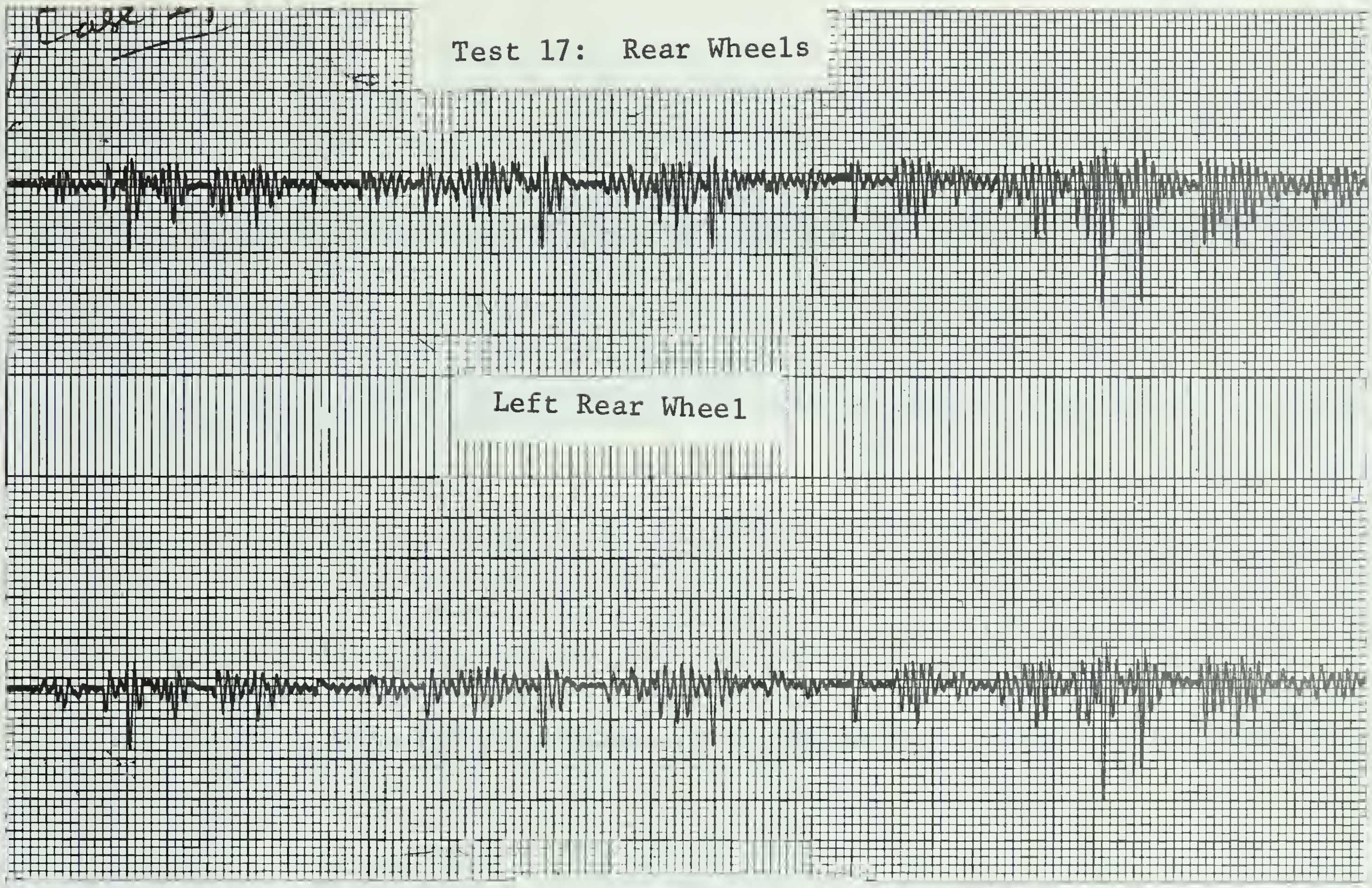
Test 16B: Left Rear Wheel and Front Axle Centre.



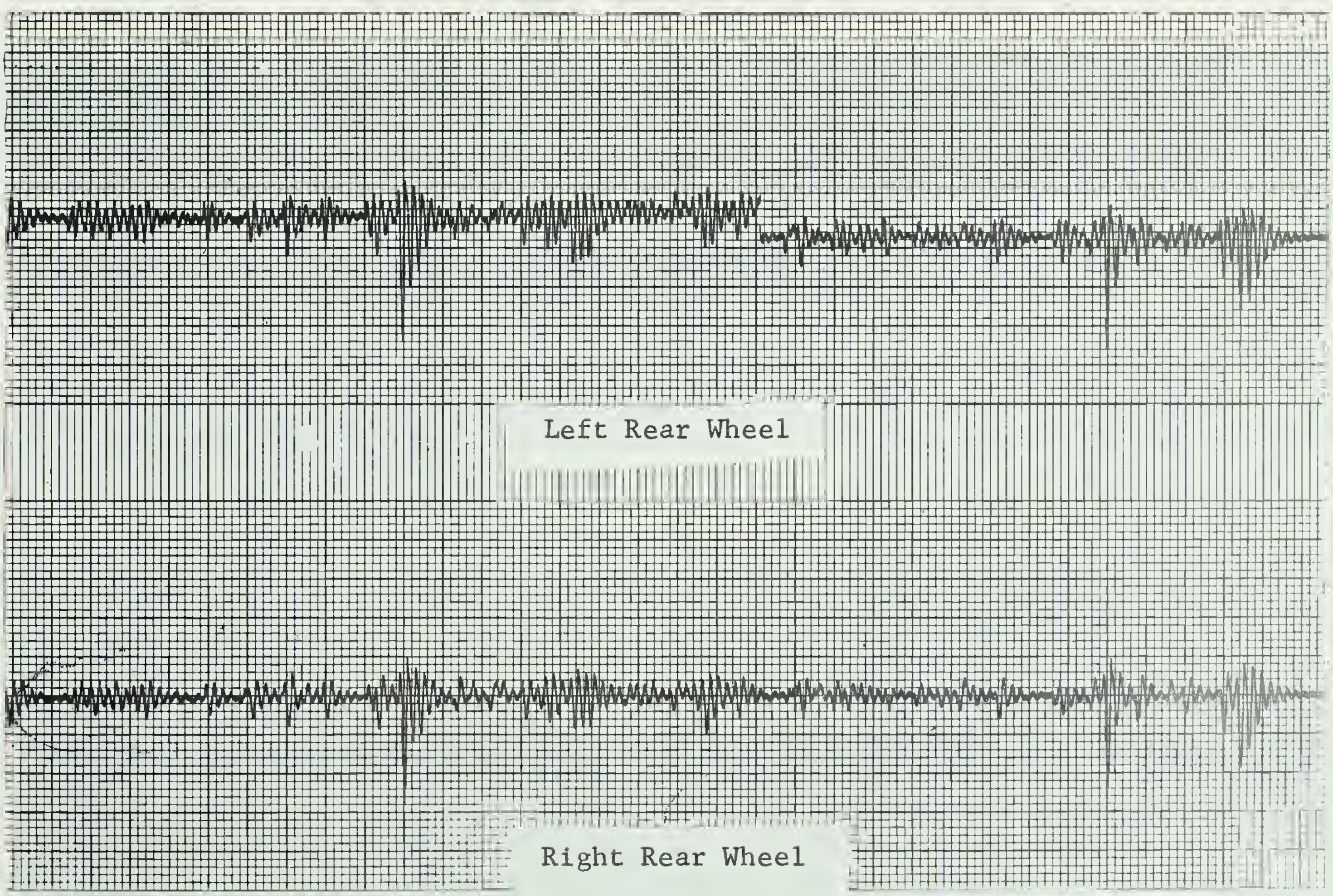
Front Axle Centre

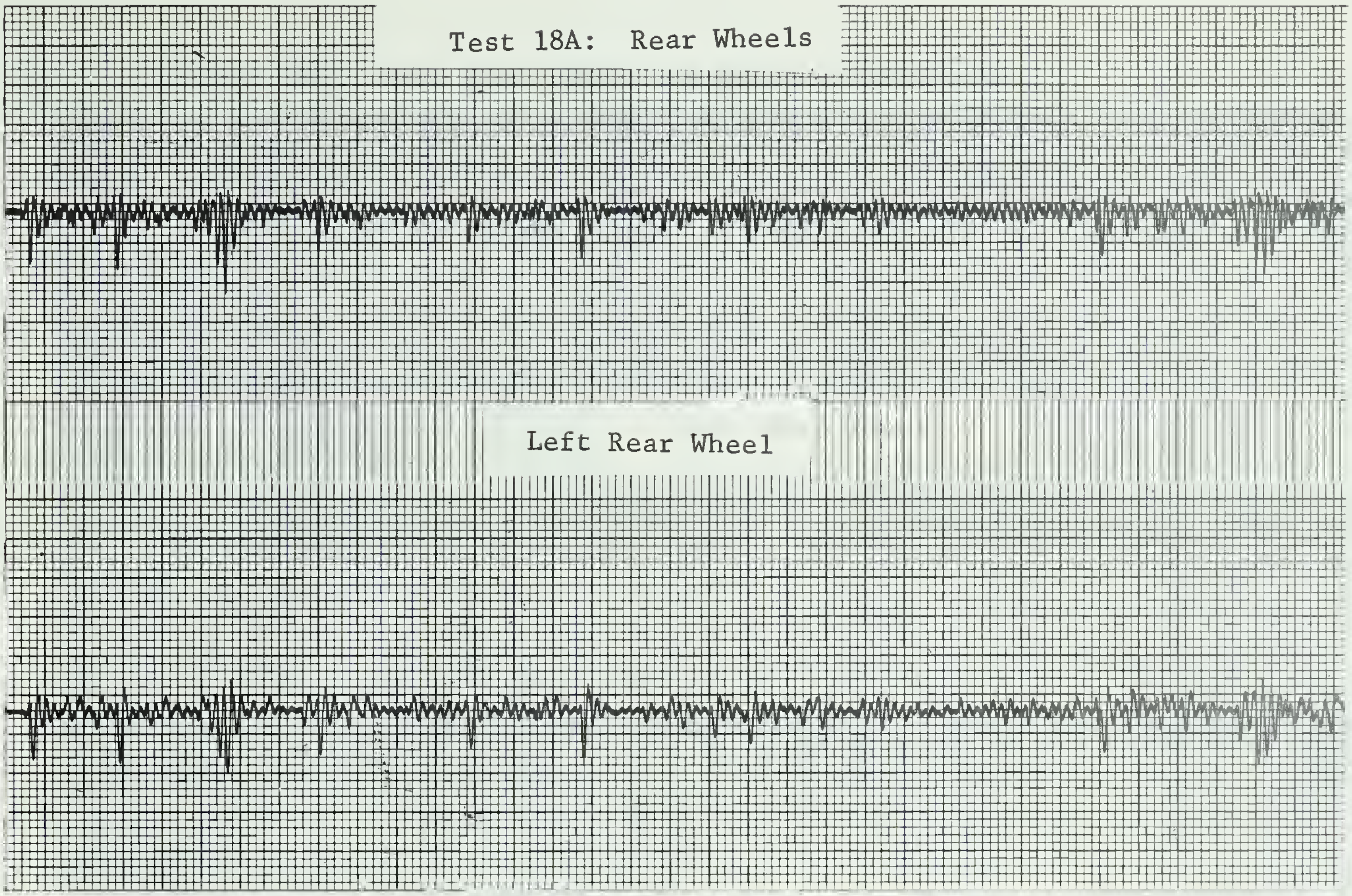


Front Axle Centre

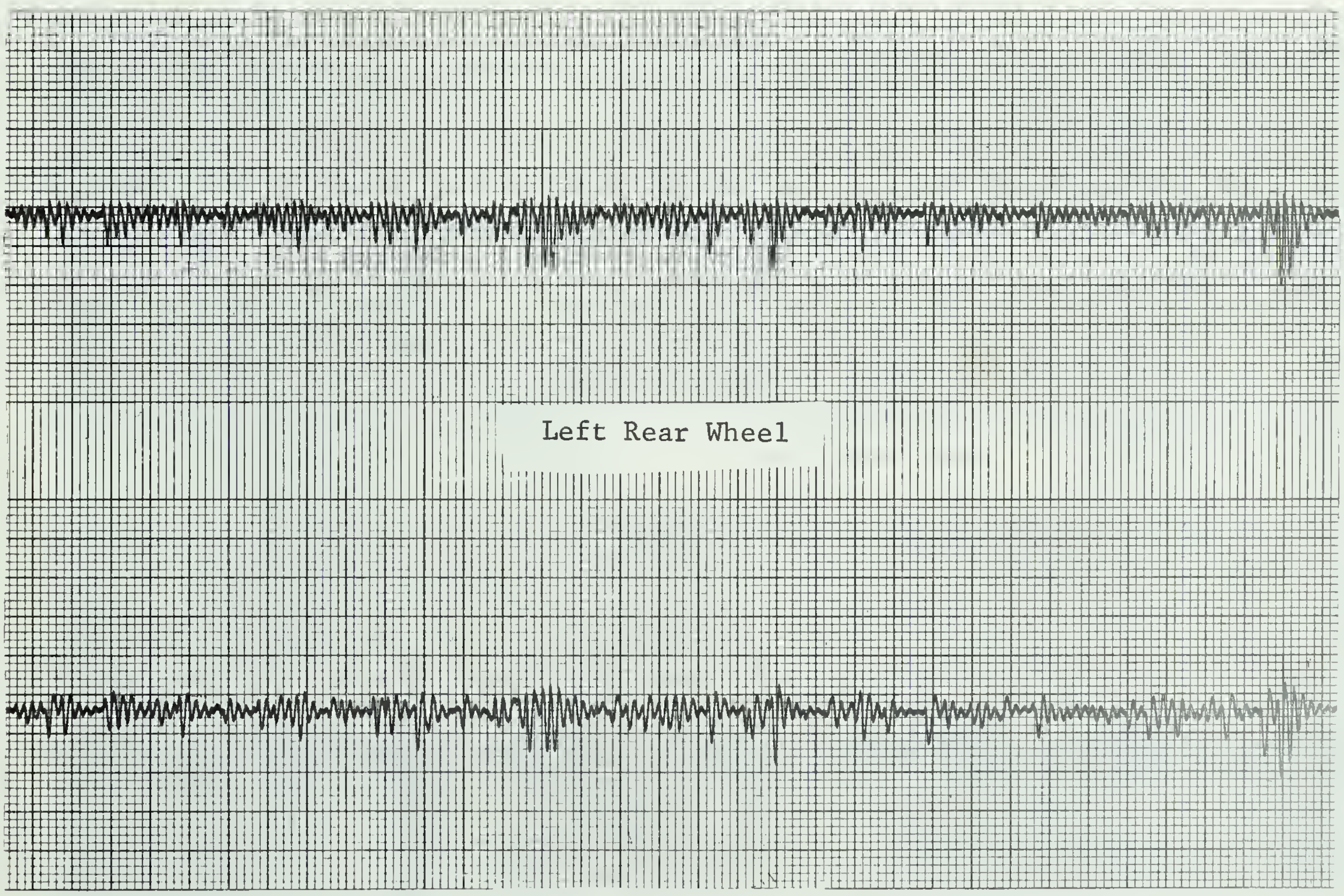


Right Rear Wheel





Right Rear Wheel



Right Rear Wheel

Test 18B: Left Rear Wheel and Front Axle Centre.